UNITED STATES DEPARTMENT OF THE INTERIOR MINERALS MANAGEMENT SERVICE

ANALYSIS AND ASSESSMENT OF
UNSUPPORTED SUBSEA PIPELINE SPANS

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An intensive research effort was performed in the area of unsupported subsea pipeline spans in order to determine the current industry "state of the art." The information resulting from this research was used to develop a method to assess and analyse pipeline free spans. This information was also used to outline preventative and corrective measures for subsea pipeline free spans. Five (5) assessment and analysis methods were developed utilizing numerous variations from different sources. Each of the five (5) methods address a particular loading on pipeline free spans. A comprehensive and orderly assessment and analysis method became available when all methods were taken into consideration simultaneously in a combined analysis method (CAM). The CAM was developed such that it could be performed by hand, if required, or with the assistance of a computerized spreadsheet. The computerized spreadsheet developed within this report allowes a single page user interface to determine the maximum allowable free span length for a given set of conditions.

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INTRODUCTION

Offshore oil and gas pipelines are being subjected to deeper water depths, more extreme environmental conditions, and harsher operating requirements than ever before. Given these conditions, free spanning pipelines are becoming more common and are often unavoidable during pipeline installation. Free spans[1] occur as a result of irregular seafloor topography at installation or during pipeline operation as a result of vibration and scour. A method of assessing and analyzing subsea pipeline free spans is an essential tool in designing new pipelines and troubleshooting existing pipelines. The purpose of this study is to develop an orderly method of subsea pipeline free span assessment and analysis, provide recommendations for pipeline free span prevention during pipeline design, and the remediation of existing pipeline free spans that pose a threat to pipeline operation.

Extensive research has provided many techniques for analyzing pipeline free spans. Five (5) assessment and analysis methods were derived from the multitude of options available from the referenced sources. These five (5) methods were the product of many variations of similar methods presented by each reference source. The methods were developed with the following objectives in mind:

- Data required for analysis is traditionally available to the pipeline design engineer
- Analysis can be performed by hand calculation without a computer
- Analysis can be performed with the assistance of a computerized spreadsheet

The analysis methods are outlined in the two (2) major categories below:

1. Static Analysis
    a. Analysis of free spans induced by low depressions
    b. Analysis of free spans using simple beam relations based on ASME B31.8 code allowables
    c. Analysis of free spans induced by elevated obstructions

[1] Unless otherwise noted all further references to free spans imply subsea pipeline free spans.

2. Dynamic (Vortex Shedding) Analysis
    a. General Vortex Vortex Induced Vibration (VIV) Analysis
    b. Analysis of Cross Flow VIV based on DNV guidelines

Three (3) methods of static analysis were developed. These analysis methods provide a comprehensive approach for assessing the two (2) major types of pipeline spans:

1. Free spans due to elevated obstructions
2. Free spans due to low depressions.

In addition, a separate static analysis was derived from simple beam relations utilizing ASME B31.8 design code allowables.

Two (2) methods of dynamic analysis were developed. Each method analyzes the effects of cross-flow vortex induced vibrations. The first method bases its analysis on the comparison of the forcing frequency with the pipeline natural frequency. The other method takes into account the latest research results used by the Det Norske Veritas (DNV) 1996 Pipeline Design Rules. The DNV method can be substantially more conservative than the other assessment and analysis methods, therefore, the consideration of this method is left to the user's discretion.

A list of design measures has been compiled in order to outline techniques of minimizing free spans of new pipelines. The list was intended for use by the pipeline engineer or designer during the planning of a pipeline project. The list outlines the appropriate information necessary to determine if a subsea pipeline free span problem exists or could be a potentional problem during pipeline operation. The list also provides recommendations for corrective actions if required.

Unanticipated pipeline free spans may have developed in existing pipelines during pipeline construction and operation. A list of measures has also been compiled for free span survey and assessment techniques and free span corrective actions.

PROJECT SCOPE

The scope of this project encompasses all of the activities necessary to understand, assess, and analyze subsea pipeline free spans. The beginning phase of the project includes an extensive research effort. This research begins with the knowledge and experience of our in-house engineering and field staff. It incorporates published industry design codes and design guidelines. The research also covers the latest information on the subject as presented in technical papers at conferences such as the Offshore Technology Conference and the Offshore Mechanics and Arctic Engineering conference. The research is further diversified with information provided by the United States Department of the Interior Minerals Management Service (MMS), Det Norske Veritas, and the United Kingdom Department of Energy.

This research yielded a wealth of information on the subject. The information provided discussions on established industry accepted methods and new research that was being performed. The investigation of new methods revealed that significant progress was made over the last few years in the area of vortex shedding analysis for subsea pipeline free spans. Much of the latest research on the subject concentrates on detailed analysis of previous methods and reconsidering previous method assumptions.

The information provided from the research efforts was then applied to the development of the following items to be delivered to the MMS upon project completion:

- Develop an orderly method of mathematically analyzing subsea unsupported pipeline spans
- Standardized free span acceptance criteria
- List pipeline design measures to minimize free span development
- List recommendations for correcting existing spans

A bulk of the project resources was devoted to the first two (2) items above. The area of static analysis remains relatively straightforward and unchanged during the last decade. Currently, there is a strong initiative within the international pipeline industry to further develop vortex shedding analysis of subsea pipeline free spans. A majority of the reference sources address vortex shedding analysis exclusively.

The last two (2) items covering preventative and corrective measures are essentially lists of standard industry practices and techniques. The lists also address the capabilities and limitations of some of the newer technology which can be used to identify free span problem areas.

SUBSEA PIPELINE FREE SPAN ASSESSMENT AND ANALYSIS

Methodology

The approach used to develop a consistent method of free span assessment and analysis was based on an analytical method that could be performed by hand and with the assistance of a computerized spreadsheet. The method was directed toward pipeline engineers and designers who have a solid understanding of submarine pipeline design and construction.

Many of the latest methods of analysis such as those introduced by the results of the MULTISPAN project[2] are based on either empirical data or require an in-depth finite element analysis (FEA). A pipeline engineer or designer typically may not have access to the required empirical data. In addition, a pipeline engineer or designer typically may not have access to FEA tools, particularly in remote field locations. Time and budget constraints may also limit accessibility to these resources.

One of the goals of this project is to make the free span analysis method as accessible as possible to the typical pipeline engineer or designer. This, in turn, will provide the MMS with a simple and highly effective method of evaluating pipeline free spans. The development of the combined analysis method (CAM) places the following criteria on variable selection:

- Consider as many variables as possible that influence free span static and dynamic response.
- Selected variables can be readily defined during typical pipeline design activities such as pre-construction route survey, geotechnical investigation, etc.
- Selected variables can be accurately estimated using published data.

Pipeline Free Span Analysis Variables

[2] Mork, K.J., Vitali, L., and Verley, R. "The MULTISPAN Project: Design Guideline for Free Spanning Pipelines." Proceedings from the 16th International Conference on Offshore Mechanics and Arctic Engineering. American Society of Mechanical Engineers. Yokahama, Japan. 1997. pg. 1.

The assessment and analysis of free spans must take into account a number of variables that can be classified into the following categories:

1. Physical and Mechanical Properties of Pipeline Materials at the Free Span
2. Physical and Mechanical Properties of Pipeline Contents at the Free Span
3. Environmental Properties around the Free Span
4. Pipeline Support and Geometric Configuration of the Free Span on the Sea Bed

These categories were derived from analysis methods developed according to the referenced sources. Each category has a particular effect on the behavior of a pipeline free span. These effects can be related to either the static or the dynamic response of the free span. Once these effects are quantified, it is possible to assess the span.

Physical and Mechanical Properties of the Pipeline Materials, such as pipe steel and concrete weight coating, influence both the static and the dynamic response of the pipeline free span. These properties include:

- Pipeline Outside Diameter
- Pipe Wall Thickness
- Young's Modulus
- Poisson's Ratio
- Specified Minimum Yield Strength of Pipe
- Concrete Weight Coating Thickness
- Density of Steel
- Concrete Weight Coating Density

The physical and mechanical properties of pipeline contents were limited to

- Density of Pipeline Contents
- Maximum Allowable Operating Pressure (MAOP)

The MAOP affects only the static response of the free span. It is assumed that this is the highest pressure to which the free span will be exposed during its operating life. This parameter is taken into consideration to determine Poisson's Effect, or the stress due to pressure shortening of the pipeline in the STATIC 2 analysis method only (See Section STATIC 2).

Density of contents affect both the static and the dynamic response of the pipeline free span. In the static case, density of the pipeline contents can affect the bending moment imposed on the free span. In the dynamic case, the density of the pipeline contents can affect the natural frequency of the free spanning pipe.

The environmental properties around the free span primarily affect the dynamic response. These properties can be the most difficult to estimate, however, they can be the most crucial in accurately predicting dynamic response of the free span. These properties include

- Density of Sea Water
- Sea Current Velocity for a 100 Year Return Period Storm
- Kinematic Viscosity of Sea Water
- Strouhal Number

The 100 year return period storm velocity is assumed to be the most severe case for the free span. Generally, the higher the current flow around the free span, the greater the probability of VIVs. The density of sea water and kinematic viscosity of sea water will vary slightly from location to location. It is recommended that the most accurate data for these variables be applied at the location of the free span under scrutiny. The Strouhal Number is a dimensionless parameter that relates the frequency of vibration to a characteristic frequency of vibration. Strouhal Number has been plotted by DNV Classification Notes No. 30.5[3] as a function of the Reynolds Number of the flow around the free spanning pipe.

Pipeline supports and sea bed geometries can be described for analysis by the following variables:

[3] DNV Classification Notes 30.5. "Environmental Conditions and Environmental Loads." Det Norske Veritas, Norway. 1991. pg. 23.

- Gap Between Pipeline and Seafloor
- Free-Span Fixity Constant
- Pipe Tension
- Damping Ratio

The gap between the pipeline and seafloor will affect the free stream velocity of the current passing around the free spanning pipe. This gap can also limit the amount of deflection that may occur due to static and dynamic loading. The fixity constant describes the boundary condition of the free span. Free spans are typically supported by boundary conditions that are neither purely simple supports nor fixed supports. Depending on the cause of the free span, stiffness of the soil, and amount of pipeline settling, the fixity constant can range between the value of 1.57 for simple supports and 3.5 for fixed supports. Pipe tension effects are considered in the STATIC 3 method only (See Section STATIC 3). In general, as pipe tension increases, the maximum allowable span length increases. The stresses on the free span due to static loading are not affected significantly by the increase in pipe tension in the STATIC 3 method. The increase in maximum allowable span length is mainly due the geometric effect of pipe straightening, which increases the touch down distance from the elevated obstruction. Estimating residual pipe tension in an already laid pipeline can be difficult. Final pipeline hookup, settling during operation, and environmental loads generally alters the pipe tension from the initial lay tension. A pipe tension value of zero is recommended unless reliable information from strain gage measurements or other sources is available. The damping ratio affects the VORTEX 2 method exclusively (See Section VORTEX 2). Free span structural damping is dependent on the pipeline material properties, boundary conditions, and fluid properties of the sea water, which affect material, Coulomb, and viscous damping effects, respectively[4].

Pipeline Free Span Assessment and Analysis Methods

No single method considers every aspect of free span behavior over the range of subsea pipeline sizes. Several loading scenarios may affect free spans in both the static and dynamic

[4] Rao, S.S. Mechanical Vibrations. Addison-Wesley Publishing Company, Inc. 1990. Second Edition. pp. 25-26.

cases. In addition, the industry has adopted several variations on analysis approaches for each of these loading scenarios. For this reason there is not one governing static analysis method nor is there one governing dynamic analysis method that applies to all ranges of pipeline and content physical properties, environmental properties, or sea bed support configurations.

The approach used in this study in analyzing free spans relied on several different methods of analysis. These methods were selected to encompass free span analysis for all ranges of pipeline and content physical properties, environmental properties, and sea bed support configurations. The methods were designed to work in unison as one combined analysis method (CAM). This CAM was developed using the following approach:

1.) Span Assessment:

The first assumption that is made for a potential or existing subsea pipeline free span is that there is a possibility that the free span is not jeopardizing the integrity of the pipeline. A potential or existing free span must be assessed in order to determine if corrective actions are necessary. The cause of the free span must also be identified in order to predict if future spans are likely to develop. If corrective actions are necessary, the amount of correction must be determined. If the cause of the free span indicates that potential future problems are likely, this may warrant preventative measures to reduce the probability of future free span development.

2.) Free Span Engineering Analysis

The focus of this report is in this area of free span engineering analysis. The free span engineering analysis is part of the assessment process. This analysis uses a mathematical approach to examine static and dynamic mechanical stresses imposed on the pipeline as a result of the free span.

3.) Free Span Acceptance Criteria

The engineering analysis results must be compared to standard acceptance criteria in order to have consistent free span evaluations. Acceptance criteria have been revised over the last several years particularly in DNV guidelines resulting from the

MULTISPAN and SUPERB projects[5,6]. Discrepancies are noted in acceptance criteria among several research papers, specifically regarding in-line VIVs.[7]

The combined analysis method (CAM) was developed for both hand calculation and computerized spreadsheet calculation. The logic used to determine the final result is the same whether it is performed by hand or by computer. The key output variable provided by the CAM is maximum allowable free span length (MAFSL). This output is based on five (5) analysis methods that were taken from selected research papers, industry publications, and design codes. The results of each of these analysis methods are compared. The comparison selects the most conservative of the MAFSLs for the methods considered. This MAFSL is reported as the final result for the CAM. All five analysis methods will not be considered simultaneously in any one case. At most four (4) of the five (5) methods will be considered

5 Mork, K.J., Vitali, L., and Verley, R. "The MULTISPAN Project: Design Guideline for Free Spanning Pipelines." Proceedings from the 16th International Conference on Offshore Mechanics and Arctic Engineering American Society of Mechanical Engineers. Yokahama, Japan. 1997. pg. 1.

6 Jiao, G., Mork, K.J., Bruschi, R., and Torbjorn, S. "The SUPERB Project: Reliability Based Design Procedures and Limit State Design Criteria for Offshore Pipelines." Proceedings from the 16th International Conference on Offshore Mechanics and Arctic Engineering, American Society of Mechanical Engineers. Yokahama, Japan. 1997. pg. 1.

7 Vitali, L., Mork, K.J., Verley, R., and Malacari, L.E. "The MULTISPAN Project: Response Models for Vortex Induced Vibrations of Submarine Pipelines." Proceedings from the 16th International Conference on Offshore Mechanics and Arctic Engineering, American Society of Mechanical Engineers. Yokahama, Japan. 1997. pg. 1.

at one time. It is the ultimate responsibility of the user to interpret which of the five (5) analysis methods govern for a given case. The interpretation of results will affect the cost of remediating an existing free span or preventing a potential free span and ensuring that the pipeline remains reliable throughout its design life. A detailed explanation of the five (5) methods follow.

FREE SPAN COMBINED ANALYSIS METHOD (CAM)

STATIC 1: Free Span Induced by Low Depressions on the Sea Floor

A low depression in the sea bed induces a free span if the natural curvature of the pipeline is unable to follow the sea bed contour. The pipeline sags at the middle of the depression which causes increased static bending stresses at the depression boundaries of a free span and at mid-span. The method is used in the CAM with the exclusion of STATIC 3: Free Spans Induced by Elevated Obstructions on the Sea Floor.

The results of dimensionless plots are used to determine the maximum allowable span lengths based on static analysis. The dimensionless plots used in this method are those developed by Mouselli[8]. These plots give a functional relationship between maximum dimensionless bending stress and dimensionless free span length. Once characteristics stresses and span lengths are determined, the MAFSL can then be calculated. The analytical expression for

$$\frac{L}{L_c} = 0.112 + 10.98\left(\frac{s_m}{s_c}\right) - 16.71\left(\frac{s_m}{s_c}\right)^2 + 10.11\left(\frac{s_m}{s_c}\right)^3, \quad 0 \le \frac{s_m}{s_c} \le 0.835$$

these plots were determined using regression curve fitting techniques in order to make them accessible to a computerized spreadsheet solution method. The regression equation is given below:

Where:

L/L_c = Dimensionless Span Length

s_m/s_c = Dimensionless Stress

[8] Mouselli, A.H. Offshore Pipeline Design, Analysis, and Methods. PennWell Publishing Co. 1981. pp. 63-64.

The static failure of a free span induced by a low depression can be due to the dead weight of the pipeline and contents causing severe bending stresses in the pipe. As the pipe sags at the middle of the depression, the pipe is uplifted on each side of the depression causing additional free spans on each side of the depression. The pipe at the depression boundary is put into severe bending relative to the adjacent pipe. The maximum bending stresses of a free span in a low depression occur at these span boundaries[9]. This methods calculates this bending stress and backs out a maximum allowable span length.

The MAFSL results of the STATIC 1 method use several assumptions. No residual axial tension is assumed in the pipe. The amount of residual tension in a pipeline is typically not available to the pipeline engineer or designer for the purpose of free span analysis. Adding residual tension effectively reduces the magnitude of static bending stress in a free span induced by a low depression. Therefore, the assumption that an axial tension of zero is a conservative assumption.

STATIC 2: Free Span Based on ASME B31.8 Code Allowables for Combined and Longitudinal Stresses

This calculation method uses longitudinal and combined stress allowables set in the Offshore Gas Transmission section of B31.8. The selection of B31.8 was arbitrary for these calculations, however this design code is representative of that used in typical pipeline designs. If required, other applicable code allowables can be substituted into this method. The use of these code allowables develops a maximum allowable free-span length by applying classical beam flexural relations to the pipe in free-span.

The longitudinal stress limit is first calculated as specified in B31.8. The longitudinal stress factor presented in Table A842.22 of 0.80 is then multiplied by the specified minimum yield strength of the free-span pipe to get the code stress limit. The maximum hoop stress is then calculated based on the maximum allowable operating pressure of the pipeline. Poisson's Effect is determined based on the maximum hoop stress. Minimum available bending stress is determined by subtracting the Poisson's Effect from the longitudinal stress limit. Two (2)

[9] Ibid., pg. 62.

resultant stresses are calculated, one for the tension side of the span and one for the compression side of the span. The minimum of the absolute values of these results is designated as the maximum available bending stress based on the longitudinal stress limit.

The combined stress limit can be based on either the Maximum Shear Stress Theory (Tresca combined stress) or the Maximum Distortional Energy Theory (Von Mises combined stress). According to B31.8 either theory can be used for limiting longitudinal stress values based on combined stress. The Von Mises combined stress is used to determine the available bending stress based on combined stress allowables. The Von Mises combined stress equation typically produces more realistic results than the Tresca combined stress equation. The longitudinal stress is solved for in the Von Mises equation as follows[10]:

$$\sigma_{Cmax} = \sqrt{\sigma_H^2 \sigma_L \sigma_H + \sigma_L^2}$$

Where:

σ_{Cmax} = Maximum Combined Stress Based on the Von Mises Equation
σ_H = Hoop Stress Limit
σ_L = Longitudinal Stress Limit

Solve for σ_L:

$$\sigma_L = \frac{\sigma_H \pm \sqrt{(\sigma_H^2)(4)(\sigma_H^2 \sigma_{Cmax}^2)}}{2}$$

Poisson's Effect is subtracted from both roots of σ_L. One root of σ_L represents the longitudinal stress in tension, and the other root represents the longitudinal stress in compression. The minimum of the absolute values of the roots determines the maximum available bending stress based on the combined stress allowable.

10 ASME B31.8-1995 Edition. "Gas Transmission and Distribution Piping Systems." The American Society of Mechanical Engineers. 1995. pg. 97.

The lesser of the combined and longitudinal stresses is taken as the maximum available bending stress. This represents the magnitude of static bending stress allowed for a pipeline free-span assuming that internal pressure effects are the only other stresses imposed on the free-span.

The pipeline free-span is assumed to be a beam of uniform cross section. A uniform transverse load which takes into account the weight of the pipe, concrete weight coating, and contents is applied. A beam of this orientation and loading is examined using the following extreme boundary conditions:

1) Simple Supports
2) Rigid Fixed Supports

If the free-span is simply supported, the maximum bending moment is located at the midpoint between the two ends of the beam. The maximum bending moment can be described by the following equation[11]:

$$M_{\mathrm{MAX}} = \frac{wL^2}{8}$$

Where:

M_{MAX}	=	Maximum Bending Moment
w	=	Uniform Transverse Load of Pipe
L	=	Length of Free Span

If the free-span is rigidly fixed at each end, the maximum bending moment is located at the fixed supports and can be described by the following equation[12]:

[11] Avallone, E.A. and Baumeister, T., III. Marks' Standard Handbook for Mechanical Engineers. McGraw Hill Book Company. Ninth Edition. 1987. pg. 5-24.

[12] Ibid., pg. 5-24.

$$M_{\mathrm{MAX}} = \frac{wL^2}{12}$$

A pipeline free-span typically has neither purely fixed ends or purely simply supported ends. The actual boundary conditions fall between these two (2) options[13]. Therefore, the boundary condition coefficients for these two (2) cases are averaged to get the resulting equation below:

$$M_{\mathrm{MAX}} = \frac{wL^2}{10}$$

The maximum bending moment equation can be also expressed in terms of pipe properties and allowable bending moment as follows:

$$M_{\mathrm{MAX}} = z\sigma_b$$

Where:

z = Pipe Section Modulus
σ_b = Maximum Allowable Bending Stress

Substitution yields:

$$z\sigma_b = \frac{wL^2}{10}$$

The maximum allowable span length is determined by solving for L in the above equation. The maximum allowable span length can be described as follows:

$$L = \sqrt{\frac{10z\sigma_b}{w}}$$

[13] Shah, B.C., White, C.N., and Rippon, I.J. "Design and Operational Considerations for Unsupported Offshore Pipeline Spans." OTC 5216. Proceedings from the 18th Annual Offshore Technology Conference. Houston, TX. 1986. pg. 5.

STATIC 3: Free Spans Induced by Elevated Obstructions on the Sea Floor

A free span will be induced on either side of an elevated obstruction on the sea floor such as a rock outcropping or man-made object. This method of calculation is based on a procedure similar to that used for STATIC 1. The use of the STATIC 3 method in the combined analysis method (CAM) excludes the use of the STATIC 1 method.

This method, like the STATIC 1 method, uses dimensionless graphs derived by Mouselli[14]. This method is more complex due to the necessity to refer to two dimensionless plots in

$$\frac{100d}{L_c} = 0.02323 + 1.251\left(\frac{s_m}{s_c}\right) + 52.18\left(\frac{s_m}{s_c}\right)^2 - 16.02\left(\frac{s_m}{s_c}\right)^3, \quad 0 \le x \le 0.405$$

order to determine a solution. In addition, residual pipe tension is given consideration using dimensionless groups.

The analysis relies on determining the height of the bottom of pipe off the sea floor. This value can be determined by measuring the height of the elevated obstruction or measuring the distance between the pipe and the sea floor using survey equipment, divers, or remote operated vehicles (ROVs). The analysis is begun by first solving for the dimensionless groups and characteristic variables. Maximum dimensionless elevation is determined using the dimensionless plot of Dimensionless Elevation vs. Maximum Dimensionless Stress. The regression equation fit for this plot is:

Where:

100d/L$_c$ = Dimensionless Elevation of Obstruction

[14] Ibid., pp. 61-64.

The Maximum Dimensionless Elevation is then plugged into the Dimensionless Span vs. Dimensionless Elevation regression equation fit:

The maximum dimensionless span length is found through interpolation based on the dimensionless pipe tension between 0 and 10. The maximum allowable span length can then be determined from the maximum dimensionless span length.

In the case of free spans induced by elevated obstructions, pipe tension has little effect on the static bending stress. The maximum static bending stress occurs at the crest of the span and is the governing stress in this case. As pipe tension increases, the pipeline touch down points on the sea bed will move further away from the elevated obstruction that is causing the span. This effectively increases the free span length. The stresses, however increase only marginally. Therefore an increase in pipe tension will cause an increase in the maximum allowable span length. A conservative assumption is to assume that the residual pipeline tension is zero.

VORTEX 1: General Vortex Induced Vibration Analysis

This calculation method addresses dynamic analysis of the pipeline free spans. Vortex-induced vibrations (VIVs) occur on pipeline free spans as well as platform riser spans

$$\frac{L}{L_c}\bigg|_{b=0} = 5.667\left(\frac{100d}{L_c}\right) - 7.600\left(\frac{100d}{L_c}\right)^2 + 3.733\left(\frac{100d}{L_c}\right)^3,\ 0 \le x \le 1$$

$$\frac{L}{L_c}\bigg|_{b=0} = 1.409 + 0.4239\left(\frac{100d}{L_c}\right) - 3.437 \bullet 10^{-2}\left(\frac{100d}{L_c}\right)^2 + 1.042 \bullet 10^{-3}\left(\frac{100d}{L_c}\right)^3,\ 0 \le x \le 7$$

$$\frac{L}{L_c}\bigg|_{b=10} = 5.150\left(\frac{100d}{L_c}\right) - 5.100\left(\frac{100d}{L_c}\right)^2 + 2.000\left(\frac{100d}{L_c}\right)^3,\ 0 < x \le 1$$

$$\frac{L}{L_c}\bigg|_{b=10} = 1.609 + 0.4740\left(\frac{100d}{L_c}\right) - 3.437 \bullet 10^{-2}\left(\frac{100d}{L_c}\right)^2 + 1.042 \bullet 10^{-3}\left(\frac{100d}{L_c}\right)^3,\ 0 < x \le 7$$

between riser clamps. This phenomenon occurs as the result of periodic shedding of vortices around the pipe. As the vortex shedding frequency approaches the pipe natural frequency, the free-span begins to resonate. This can result in rapid pipeline failure. It is recognized that there is a wide range of vortex shedding frequencies that induce excessive stresses, which may cause structural fatigue damage to the pipe and possible failure. A pipeline has very little natural damping. This further magnifies VIV effects.

The analysis is based on maintaining the reduced velocity around the pipeline less than 3.0-5.0. This corresponds to the onset of cross flow (VIV)[15]. The reduced velocity is given by:

Where:

V_R = Reduced Velocity of the current flow around the pipeline

U = Sea Current Velocity at the Pipeline Span

f_n = Pipe Natural Frequency

D_{TOT} = Overall Outside Diameter of the Pipeline at the Free Span

$$V_R = \frac{U}{f_n D_{TOT}}$$

15 Mork, K., and Vitali, L. "An Approach to Design Against Cross-Flow VIV For Submarine Pipelines." Dynamics of Structures. Aalborg University, Denmark. 1996. pg. 1.

The value for reduced velocity typically corresponds to the following relation[16]:

$$f_s < 0.7 f_n$$

Where:

f_s = Vortex Shedding Frequency

Therefore if the vortex shedding frequency around the pipeline free span is maintained at less than 70% of the natural frequency of the pipeline, the probability of cross flow VIV will be minimized. The pipeline natural frequency can be described using the following equation[17]:

Where:

$$f_n = \frac{kp}{2L^2}\sqrt{\frac{EI}{M}}$$

$\frac{kp}{2}$ = C = Free Span End Fixity Constant

EI = Free Span Pipe Stiffness

M = Dynamic Mass of Submerged Pipe

L = Free Span Length

16 Mouselli, A.H. Offshore Pipeline Design, Analysis, and Methods. PennWell Publishing Co. 1981. pp. 50-52.

17 Nielsen, R., and Gravesen, H., edited by de la Mare, R.F. Advances in Offshore Oil and Gas Pipeline Technology. Gulf Publishing Company. Houston TX, 1985. pg. 326.

The free span end fixity constant, C, is generally taken as 1.54 for simply supported ends and 3.50 for fully fixed ends. In an actual pipeline, free span end fixity will neither be purely fixed or purely simply supported. Rather, the free span ends will be partially fixed. It is recommended that the end fixity constant be taken somewhere between these two values in order to approximate actual free span end conditions[18]. The final solution can be determined by substitution as follows:

$$f_s = (0.7)\frac{C}{L^2}\sqrt{\frac{EI}{M}}$$

The maximum allowable span length can be determined by solving for *L* in the above equation.

VORTEX 2: Cross-Flow Vortex Induced Vibration Analysis

This method of free span analysis is based on limit state and partial safety factor design criteria. DNV adopted this methodology in its 1996 Pipeline Design Rules. This method was derived from research performed on the MULTISPAN project and is used as a basis for the DNV Design Guideline[19].

Consideration of this method as part of the CAM should be evaluated on a case-by-case basis. This method can be significantly more conservative than the static cases and VORTEX 1.

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18 Shah, B.C., White, C.N., and Rippon, I.J. "Design and Operational Considerations for Unsupported Offshore Pipeline Spans." OTC 5216. Proceedings from the 18th Annual Offshore Technology Conference. Houston, TX. 1986. pg. 5.

19 Mork, K., and Vitali, L., "An Approach to Design Against Cross-Flow VIV For Submarine Pipelines." Dynamics of Structures. Aalborg University, Denmark. 1996. pg. 1.

The implementation of this method requires several variables to be defined. The stability parameter is a function of the structural damping ratio, dynamic mass, and outside diameter of the pipeline[20]:

$$K_s = p^2 z\left(\frac{r_{os}}{r} - C_m\right)$$

Where:

z = Ratio of Structural Damping

r_{os}/r = Specific Mass

[20] Vitali, L., Mork, K.J., Verley, R., and Malacari, L.E. "The MULTISPAN Project: Response Models for Vortex Induced Vibrations of Submarine Pipelines." Proceedings from the 16th International Conference on Offshore Mechanics and Arctic Engineering. American Society of Mechanical Engineers. Yokahama, Japan. pg. 5.

C_m = Added Mass Coefficient[21]

The ratio of structural damping is a function of the logarithmic decrement coefficient of structural damping, δ [22]:

Strouhal number is a function of Reynolds number and the distance between the free span and the sea floor (indicated by the gap ratio). Data from the MULTISPAN project allowed a linear relationship between Strouhal number and the gap ratio as follows[23]:

$$d = \frac{2pz}{\sqrt{1-z^2}}$$

$$S_t = 0.27 - 0.03(e/D_{TOT})$$

[21] DNV Classification Notes 30.5. "Environmental Conditions and Environmental Loads." Det Norske Veritas, Norway. March 1991. pg. 23.

[22] Rao, S.S. Mechanical Vibrations. Addison-Wesley Publishing Company, Inc. 1990. Second Edition. pg. 89.

[23] Vitali, L., Mork, K.J., Verley, R., and Malacari, L.E. "The MULTISPAN Project: Response Models for Vortex Induced Vibrations of Submarine Pipelines." Proceedings from the 16th International Conference on Offshore Mechanics and Arctic Engineering, ASME, Yokahama, Japan. pg. 5.

Where:

e/D_{TOT} = Gap Ratio

The limit state based partial safety factor design is achieved by relating the pipeline natural frequency to the reduced velocity of the flow around the pipe. The reduced velocity is limited to the onset of cross-flow VIV. Several partial safety factors are employed in order to ensure that cross flow VIV does not occur. The limit state equation that limits the onset of cross-flow VIV is:

$$f_n \geq \frac{U}{V_{R,onset} D} g_T \Psi_D \Psi_R \Psi_U$$

Where:

f_n = Pipeline Natural Frequency

U = Sea Current Velocity

D = Pipeline Outside Diameter

g_T = Safety Class Factor

Y_D = Period Transformation Factor

Y_R = Natural Frequency Reduction Factor

Y_U = Extreme Current Variability Factor

The period transformation factor is related to the time it takes current induced VIV to reach full amplitude of vibration[24]. The typical recommended value for this partial safety factor is 1.0. The natural frequency reduction factor is normally set to 1.0, however this value may be taken as 0.9 if the natural cross-flow frequency is well defined. The extreme current variability factor is also normally set to 1.0. If a large current variability is expected in the area of the free span, then this

[24] Mathiesen, M., Hansen, E. A., Andersen, O.J., and Bruschi, R. “The MULTISPAN Project: Near Seabed Flow In Macro-Roughness Areas.” Proceedings from the 16th International Conference on Offshore Mechanics and Arctic Engineering. American Society of Mechanical Engineers. Yokahama, Japan. 1997. pg. 20.

factor should be set to 1.1 for the normal and high safety classes.[25]. Three (3) safety classes are defined in this method. The selection of safety class governs the value of the safety class factor. If the safety class factor is designated "Low" for temporary conditions, the value of γ_T is 1.7. "Normal" and "High" safety class values would be 2.0 and 2.3, respectively. The "Normal" and "High" safety classes apply to in-service pipelines[26].

The determination of the pipeline natural frequency requires the use of a finite element analysis model[27]. The method selected in this report to determine the natural frequency of the pipeline is the natural frequency equation used in the VORTEX 1. This method offers an approximate solution that enables the user to quickly achieve results by performing relatively simple hand calculations. The resulting solution is achieved by solving the following equation for the maximum allowable span length, *L*:

$$\frac{kp}{2L^2}\sqrt{\frac{EI}{m}} = \frac{U}{V_{R,onset}D} g_T \Psi_D \Psi_R \Psi_U$$

[25] Mork, K.J., Vitali, L., and Verley, R. "The MULTISPAN Project: Design Guideline for Free Spanning Pipelines." Proceedings from the 16th International Conference on Offshore Mechanics and Arctic Engineering. American Society of Mechanical Engineers. Yokahama, Japan. 1997. pg. 5.

[26] Ibid., pg. 5

[27] Mork, K.J., Vitali, L., "An Approach to Design Against Cross-Flow VIV for Submarine Pipelines." Proceedings from the 16th International Conference on Offshore Mechanics and Arctic Engineering. American Society of Mechanical Engineers. Yokahama, Japan. 1997. pg. 5.

In-Line VIV vs. Cross-Flow VIV

All of the analyses described above are subject to the limitations described by their corresponding reference sources. These limitations are imposed on the use of the methods herein due to the admitted lack of data by the sources referenced. The limitations imposed by several of the sources, particularly in the analysis of in-line VIV, has placed this method of analysis beyond the scope of this project. There is a great disparity of opinion regarding the effects of in-line VIV. The disagreement lies primarily on establishing consistent acceptance criteria for this phenomenon. Further experimental research may be required in order to establish a consistent assessment and analysis method for in-line VIV.

In order to understand the reasoning behind these disagreements, it is necessary to understand the mechanism that causes this effect. As current velocity increases across a free spanning pipeline, the onset of in-line VIV will occur at a specific reduced velocity. In-line VIV is the vibration of pipe in the same plane as the current flow. In-line VIV occurs in two distinct instability regions. In the first instability region, the amplitude of in-line VIV tends to increase as reduced velocity increases. Typical values of reduced velocity marking the first instability region are $1.0<V_R<2.2$. As the current velocity continues to increase, the amplitude of vibration no longer increases, signifying the second instability region. The second instability region typically occurs when $2.2<V_R<4.5$. Cross-flow VIV can begin at reduced velocities between 3.0 and 5.0. Cross-flow oscillations are free span vibrations that occur in the plane perpendicular to that of the current flow. The amplitude of in-line VIV is approximately one tenth that of the corresponding amplitude of cross-flow vibration in most cases[28]. In some cases it is difficult to determine the transition point between in-line second stability region regimes and the onset of cross-flow vibrations[29]. Failure due to cross flow vibrations can occur within just a few cycles, whereas failure time due to in-line VIV may exceed the design life of the pipeline. Some sources discount the need to examine the effects of in-line VIV[30]. Other sources,

[28] Mork, K.J., Vitali, L., and Verley, R. "The MULTISPAN Project: Design Guideline for Free Spanning Pipelines." Proceedings from the 16th International Conference on Offshore Mechanics and Arctic Engineering. American Society of Mechanical Engineers. Yokahama, Japan. 1997. pg. 5.

[29] Vitali, L., Mork, K.J., Verley, R., and Malacari, L.E. "The MULTISPAN Project: Response Models for Vortex Induced Vibrations of Submarine Pipelines." Proceedings from the 16th International Conference on Offshore Mechanics and Arctic Engineering, American Society of Mechanical Engineers. Yokahama, Japan. 1997. pg. 2.

[30] Tsahalis, D.T., and Jones, W.T. " The Effect of Sea-Bottom Proximity on the Fatigue Life of Suspended Spans of Offshore Pipelines

such as those relating to the MULTISPAN project have attempted to define a standard analysis procedure based on fatigue considerations[31].

Undergoing Vortex-Induced Vibrations." OTC 4231. Proceedings from the 14th Annual Offshore Technology Conference. Houston, TX. 1982. pg. 1.

[31] Mork, K.J., Vitali, L., and Verley, R. "The MULTISPAN Project: Design Guideline for Free Spanning Pipelines." Proceedings from the 16th International Conference on Offshore Mechanics and Arctic Engineering, American Society of Mechanical Engineers. Yokahama, Japan. 1997. pg. 3.

One option to control the effects of in-line vibration is to prevent the reduced velocity from reaching the on-set value for in-line vibration. Since the on-set value is a function of the free span natural frequency, reduced velocity can be controlled by limiting the free span length. However, a design which limits the free span length to the on-set of in-line VIV can be impractical from a practical and economic standpoint[32].

This project does not consider the effects of in-line VIV due to the following reasons:

- The latest published papers have noted that there is not a comprehensive understanding on the subject when flow direction, free stream turbulence, and sea bottom proximity is taken into account. Contradictory results are noted between experimental models from MULTISPAN project and earlier experimental models by others[33].
- Assessment and analysis methods for in-line VIV presented by the reference sources provided widely varying acceptance criteria.

Determining Combined Analysis Method Governing Equations

All of the calculation methods described in the above sections can be found in Appendix C with sample calculations of each method. Appendix A contains the hand calculations necessary to utilize the combined analysis method (CAM). The interpretation of the results from the CAM is discussed in further detail below.

The combined analysis method provides five (5) methods to compute the maximum allowable free span length for a given pipeline free span. Each of these methods consider specific loading on the free span. When assessing a free span, the pipeline status must be taken into consideration such as:

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[32] Tassini, P.A., Lolli A., Scolari, G., Mattiello, D., and Bruschi, R. "The Submarine Vortex Shedding Project: Background, Overview, And Future Fall-Out on Pipeline Design." OTC 4231. Proceedings from the 21st Annual Offshore Technology Conference. Houston, TX. 1989. pg. 4.

[33] Vitali, L., Mork, K.J., Verley, R., Malacari, L.E. "The MULTISPAN Project: Response Models for Vortex Induced Vibrations of Submarine Pipelines." Proceedings from the 16th International Conference on Offshore Mechanics and Arctic Engineering. American Society of Mechanical Engineers. Yokahama, Japan. 1997. pg. 2.

1. Installation in Progress
2. Installed and Awaiting Tie-In to Risers or Other Pipelines.
3. Fully Operational

Other factors such as the age of the pipeline, soil conditions surrounding the pipe, and burial depth may determine which method will govern in a situation. If a span exists in very deep water where there is virtually no currents, it may be adequate to design the pipeline based on static analysis alone. If the pipeline is in close proximity to populated areas and may present a danger to public safety, it may be appropriate to consider using the DNV method described in VORTEX 2.

The Combined Analysis Method (CAM) recommends a procedure to assist in determining the governing method. The pipeline engineer or designer has the ultimate responsibility of chosing the governing method to analyze a given pipeline free span and should consider the following questions:

1.) Consider the 1996 DNV design guidelines developed in the MULTISPAN project?

The answer to this question will determine if the result from the VORTEX 2 method is to be considered in the CAM. If the answer is no, the VORTEX 2 method does not have to be considered.

2.) Is the free span induced by an elevated obstruction or a low depression on the sea floor?

If the free span is induced by an elevated obstruction, the STATIC 1 case can be eliminated from the CAM. If the free span is induced by a low depression then the STATIC 3 case can be eliminated from the CAM.

Once the relevant methods of analysis are chosen, the results from the each of the methods can be computed and compared. The maximum allowable free span length (MAFSL) can be determined from the minimum values among the MAFSLs for each of the applicable methods. This minimum MAFSL should be used as a guide only. If possible, the values should be compared to other case studies to determine if this is a reasonable result. In many cases, it may be appropriate to increase the MAFSL if other information warrants such an adjustment.

SPREADSHEET ANALYSIS

In addition to the hand calculation method, a spreadsheet calculation method has been developed to perform the functions of the hand calculations. The calculations performed by the spreadsheet are organized similarly to the hand calculations for ease of reference. The spreadsheet is provided in Appendix B. This spreadsheet presents values that are pre-input to match the hand calculation case provided in Appendix C.

REFERENCE GUIDE Worksheet

The spreadsheet is designed in several different worksheets beginning with the REFERENCE GUIDE worksheet. The REFERENCE GUIDE is an abbreviated set of instructions that describes the spreadsheet operations and introduces the other worksheets and variables. It also recommends values to several of the property variables that are encountered in the analysis process.

MASTER Worksheet

The MASTER worksheet functions as the input/output page of the spreadsheet. The user of the spreadsheet should enter values for the analysis variables only on this worksheet in the boxes provided. These input variables are transferred into each of the individual analysis methods as required. The MASTER worksheet compares all of the answers from the applicable analysis methods and returns the value of the MAFSL according to the procedure described in the Governing Equations section above. The MASTER worksheet also performs some error checking to ensure that all of the required parameters are entered. It also performs checks to ensure that some of the variables are within an acceptable range for analysis. A flowchart is provided to illustrate the logic used in the MASTER worksheet. This flowchart can be found in the REFERENCE GUIDE worksheet in Appendix B.

GLOBAL VARIABLES Worksheet

This worksheet summarizes the input of variables used in several different methods as part of the Combined Analysis Method (CAM). The applicable variables from this worksheet are supplied to the Global Variable sections of the individual analysis worksheets. This worksheet is meant to be a ***read-only*** worksheet. The variables described here either derive their input from the MASTER worksheet or are calculated internally. The variables that are internally calculated use the input

provided from the MASTER worksheet as required. An example of a calculated variable is Pipe Inside Diameter, D_i, which is a function of the Pipe Outside Diameter, D_o, and Pipe Wall Thickness, t. Both D_o and t are supplied to the GLOBAL VARIABLES Worksheet from the MASTER Worksheet. The MASTER Worksheet Calculates
D_i using the equation:

$$D_i = (D_o - 2t)$$

Analysis Method Worksheets

The analysis method worksheets consist of the STATIC 1, STATIC 2, STATIC 3, VORTEX 1, and VORTEX 2 worksheets. These worksheets are also meant to be ***read-only*** as is the case with the GLOBAL VARIABLES worksheet. These worksheets automate the hand calculation procedures provided in Appendix C. The worksheets were created for quick access to the results of each analysis method as well as provide key variable calculations within each method. The results from each of the analysis methods should be taken into consideration in the determination of the most practical method to use for a given case.

FREE SPAN PREVENTION

There are various tools available to the pipeline engineer or designer that enables them to predicted potential pipeline free spanning areas before the pipeline is installed. These tools in conjunction with engineering analysis of the information they provide have been highly effective in preventing free spans during installation as well as future development during pipeline operation. The recommendations presented below have been provided by our highly experienced staff members that specialize in pipeline pre-lay survey and route selection:

- Perform a thorough hazard survey along the pipeline route including a side scan survey in conjunction with a subbottom profile. This will provide comprehensive information on the existing bottom conditions. Engineering analysis may be required to determine if bottom conditions will induce a span in the proposed pipeline.

- Review hazard survey data as soon as possible, preferably in the field, while data acquisition is still in progress. If areas of potential spanning are identified along the centerline of the route, check to see if the route centerline could be moved within the right of way (ROW) or within the surveyed area. If this is not achievable, additional survey may be required. The advantage of reviewing data in the field is that the survey area can be actively changed if spanning problems develop within the original survey area. This would minimize surveying of unsuitable areas and prevent potential mobilization and demobilization costs from being incurred for an additional survey. For example, in areas of expected rough bottom features, the pre-plot of survey lines should be prepared for rapid expansion or adjustment while data is being gathered in the field.

- Subbottom profilers and side scan sonar can miss small isolated hard bottom features that can cause spanning problems for pipelines. This can be remedied by further interpretation of data by experienced geologists and engineers or by performing an ROV inspection.

- Swath bathymetry data can be used as a supplement to side scan data to identify possible spanning problems. Swath bathymetry can provide high resolution depth contours in shallow water, however it can miss small bottom features in deeper water.

- Minimize pipe lay tension by specifying a minimum and maximum tension allowed for pipeline construction.

- Strategically locate points of intersection (P.I.s) along the pipeline route to minimize the chance of dragging the pipeline over a bottom feature during installation.

- If a a known area of scour lies along the pipeline route and is unavoidable, concrete mats in conjunction with geotechnical fabric can be placed in the potential scour area. This will prevent the subsequent formation of spans due to scour during pipeline operation.

FREE SPAN REMEDIATION

In some cases, free spans simply cannot be prevented due to sea floor topography. Pipeline spans often develop after a pipeline is installed and even during pipeline operation. The free span should be first assessed and analyzed in order to determine the magnitude of correction required for safe and reliable pipeline operation. Once the amount of correction has been established, the following recommendations can be used to remediate the free-span. These recommendations were provided by our in-house staff specializing in pipeline construction and pipeline repair:

- Jetting can alleviate spanning problems due to soil ripples and sand waves. The pipeline is essentially lowered by excavation until the span is eliminated. When lowering the pipeline is impractical it may be necessary to use diver placed sand/cement bags, grout bags, or in extreme conditions concrete blocks to support the spanning pipe.

- It may be required to physically move the pipeline away from a bottom feature when it is impractical or unfeasible to lower or support the pipeline around the feature

- If an area of scour is discovered after the pipeline is laid, first remediate any resultant spans using the methods listed above. Concrete mats in conjunction with geotechnical fabric can be placed in the area subject to scour to prevent the subsequent formation of spans.

CONCLUDING REMARKS

This report has covered the methods required to assess and analyze submarine pipeline free spans. The assessment and analysis methods were based on information that has been published over the last two (2) decades. These sources have originated globally from societies, joint industry ventures, published codes, and independent authors. The information was used to develop a Combined Analysis Method (CAM), which consisted of five (5) individual analysis methods. Each of these methods concentrate on a specific loading scenario. The results of each method are compared in order to determine the governing method for each particular case and, ultimately, the maximum allowable free span length for a given case. The procedure for performing the CAM has been provided in two (2) formats: 1.) Hand Calculation and 2.) Computerized Spreadsheet. The methods used in both formats are identical. It is important to note that the CAM is intended to be used by engineers and designers who are experienced in the area of submarine pipeline design and construction. The user of this method must not rely solely on the MAFSL generated as the final result on the MASTER spreadsheet. This result should only be used as a guide in determining the most practical and reasonable MAFSL for a given case. It is the responsibility of the user to review each of the five (5) methods on an individual basis to determine the MAFSL based on sound engineering knowledge and experience.

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11.) Nielsen, R. And Gravesen, H., edited by de la Mare, R.F. Advances in Offshore Oil and Gas Pipeline Technology. Gulf Publishing Company. Houston, TX. 1985.

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13.) Tsahalis, D.T. and Jones, W.T. "The Effect of Sea-Bottom Proximity on Fatigue Life of Suspended Spans of Offshore Pipeline Undergoing Vortex-Induced Vibrations." OTC 4231. Proceedings from the 14th Annual Offshore Technology Conference. Houston, TX. 1986.

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Additional Source Used in GLOBAL VARIABLES Example Calculation

1.) Fox, Robert W. and McDonald, Alan T. Introduction to Fluid Mechanics. John Wiley and Sons, Inc. Third Edition. 1985.

APPENDIX A:

Comparison of CAESAR II Model Results with Combined Analysis Method (CAM) Results

COMPARISON OF CAESAR II MODEL RESULTS WITH COMBINED ANALYSIS METHOD RESULTS

Several different checks were performed to insure that the results from the calculation methods described here were reasonable and practical. The checks were based on the experience and knowledge that is available in-house as well as thorough checking of hand calculations and analysis methods. As a final overall check, several computer stress analysis models were created in CAESAR II. These computer stress analysis models were used as a comparison to the Static 1 and Static 2 calculation methods outlined in the Combined Analysis Method (CAM). CAESAR II is a commercial software package used to model process piping as well as buried and unburied pipelines. The CAESAR II models considered stress allowables based on ANSI/ASME B31.8 code requirements for the static case only. That is, the models only considered dead weight and static pressure effects in accordance with the code. A total of five (5) models were created for comparison purposes. The models and results are summarized in the table below:

							CAESAR II	STATIC 1	STATIC 2
Case 1	D_o	t	t_c	ρ_{ocn}	S_y	P	L	L	L
Case 2	2.875*	0.276"	0	64 lbs/ft^3	35 ksi	2775 psi	70 ft.	86 ft.	54 ft.
Case 3	8.625"	0.500"	0	64 lbs/ft^3	35 ksi	2775 psi	110 ft.	151 ft.	49 ft.
Case 4	12.75"	0.688"	0	64 lbs/ft^3	35 ksi	1800 psi	150 ft.	182 ft.	109 ft.
Case 5	18.00"	0.562"	1.5"	64 lbs/ft^3	52 ksi	1800 psi	190 ft.	222 ft.	113 ft.
Case 6	30.00"	0.562"	2.75"	5 lbs/ft^3	65 ksi	1800 psi	245 ft.	833 ft.	155 ft.

* Non-Standard pipe size

This table illustrates that the CAESAR II case results generally falls between the results for the Static 1 and Static 2 cases for a wide range of input parameters. The input parameters were chosen in order to test a variety of spreadsheet input variables and to verify that the calculation methods work for a wide range of pipe sizes. The table also illustrates that each of the individual methods (Static 1, Static 2, etc.) do not necessarily work for the entire range of pipe sizes. A logical comparison using the Combined Analysis Method (CAM) among all five (5) methods presented here overcomes this limitation. The CAM provides a comprehensive and systematic way of determining maximum allowable pipeline span for all common subsea pipeline sizes and environmental conditions.

APPENDIX B:

Example Spreadsheet Calculations

Minerals Management Service
Assessment and Analysis of Unsupported Subsea Pipeline Spans

REFERENCE GUIDE

This spreadsheet is submitted in conjunction with the main report on the Assessment and Analysis of Unsupported Subsea Pipeline Spans. The results of this spreadsheet are intended to be interpreted in accordance with the theories and methods presented in the main report.

The spreadsheet is divided into eight (8) worksheets including this REFERENCE GUIDE Worksheet. These worksheets include a MASTER Worksheet and six (6) calculation worksheets. The MASTER Worksheet is a single-page user interface instrument. This quick reference guide is intended to provide information on the general use of the MASTER Worksheet. The worksheets titled GLOBAL VARIABLES, STATIC 1, STATIC 2, STATIC 3, VORTEX 1, and VORTEX 2 are designed to be read-only and derive their input from the MASTER Worksheet The functions of each worksheet are listed below:

MASTER Worksheet:	User interface input/output page.
GLOBAL VARIABLE Worksheet:	Variable calculations that appear on more than one of the following worksheets.
STATIC 1 Worksheet:	Static analysis for low depression induced free spans.
STATIC 2 Worksheet:	Static analysis based on B31.8 longitudinal and combined stress allowables. Pipe is considered as a semi-fixed beam.
STATIC 3 Worksheet:	Static analysis for elevated obstruction induced free spans.
VORTEX 1 Worksheet:	Dynamic analysis for general vortex induced vibration of free spans.
VORTEX 2 Worksheet:	Dynamic analysis for cross-flow vortex induced vibration based on the results of the MULTISPAN project.

The function of GLOBAL VARIABLES, STATIC 1, 2, 3, VORTEX 1, and 2 is to calculate the local maximum allowable free-span lengths (MAFSLs) for each method described in the main report. The results of all applicable worksheets are compared, and the most conservative result is reported on the MASTER Worksheet as the overall MAFSL. This procedure is known as the Combined Analysis Method (CAM). In order to protect against over-conservatism, the user is prompted to select between several input alternatives in the MASTER Worksheet. These alternatives are described in detail below. The user also has the option to view the results of the individual module worksheets, i.e. STATIC 1, STATIC 2, etc., if required.

GENERAL INFORMATION

Consider 1996 DNV Guidelines Developed within the MULTISPAN project:

Check this box if consideration will be given to the DNV Guidelines developed within the MULTISPAN project. The VORTEX 2 Worksheet calculates the local MAFSL based on this method. A more detailed discussion of the MULTISPAN project can be found in the main report along with a list of references. Consideration of these guidelines produces results that can to be more conservative than U.S. standards. Therefore, if specifications allow the pipeline to be exempt from DNV Guidelines then this option can be left un-checked.

The free-span is induced by ELEVATED OBSTRUCTION or LOW DEPRESSION:

This option selects between the STATIC 1 AND STATIC 3 calculation methods. These selections are mutually exclusive. These worksheets will be used in addition to the other worksheets that apply, i.e. STATIC 2, VORTEX 1, etc. The local MAFSL calculated by the STATIC 1 and STATIC 3 worksheets typically do not govern the overall result. Therefore, the overall MAFSL may be the same regardless of this selection.

REQUIRED INPUT VARIABLES:

The following variables are required for proper spreadsheet operation. It is asumed that the pipline has been properly designed for required internal flow rates, internal pressure, on-bottom stability, etc.

D_0 : Actual pipeline O.D. in inches
t : Actual pipeline wall thickness in inches
E : Modulus of Elasticity of the Pipeline in psi -- Typical range for carbon steel: 29,000,000 psi - 30,000,000 psi
ν_0 : Poisson's Ratio of the Pipeline -- Typical range for steel: 0.26 - 0.30
S_y : Specified Minimum Yield Strength of Pipe in psi -- refer to applicable design codes or specifications
t_c : Concrete Weight Coating Thickness in inches
ρ_{os} : Density of Pipeline Steel in lbs./ft.^{3}
ρ_{oc} : Concrete Weight Coating Density in lbs./ft.^{3}
ρ_{ocn}: Density of Pipeline Contents in lbs./ft.^{3}
U : Sea Current Velocity for a 100 year Return Period Storm in ft./s can be obtained from an oceanographic survey at the free-span location. It is important to determine the sea current at the approximate elevation with respect to the sea floor and location of the pipeline free-span. Data is available through meteorological and oceanographic consulting firms for many areas around the world.

ν_k : Kinematic Viscosity of Sea Water -- This value is typically given as 1.13 X 10^{-5} $ft.^2/s$ at 70° F (20° C). This value can vary with temperature and sea water composition.

e : This is the maximum gap between the free-span and the sea floor in feet. This distance can be estimated using a known bottom profile or can be measured by diver or ROV survey.

P_{maop}: Maximum Allowable Operating Pressure in psi

C : Free-Span Fixity Constant -- This is a boundary condition constant for the pipeline free-span. A free-span that is rigidly supported at each end would have a fixity constant of 3.53. If the free-span is simply supported, the fixity constant would be $\pi/2$ or approximately 1.57. Typically, pipeline free-span supports are not purely fixed nor simply supported but somewhere in between. Therefore, these two values are averaged, which yields a fixity constant of 2.55.

DNV GUIDELINE-SPECIFIC INPUT VARIABLES:

This set of input parameters applies only if the 1996 DNV Guidelines are taken into consideration. If so, these parameters are required inputs:

C_m : Added Mass Coefficient -- This value is defined by DNV Classification Notes No. 30.5 titled, "Environmental Conditions and Environmental Loads." For a cylindrical object subject to cross-flow motion, this value is 1.0.

ψ_R : Natural Frequency Reduction Factor -- This factor is normally set to 1.0, however it may be taken as 0.9 if the natural cross-flow frequency is well defined.

ψ_U : Extreme Current Variablilty Factor -- This factor is normally set to 1.0 but should be taken as 1.1 in case of a large variability in the extreme current velocity for safety classes NORMAL and HIGH.

ψ_D : Period Transformation Factor -- This factor accounts for the time averaged periods it takes the pipeline to reach full vibrational amplitude. This factor can be taken as 1.0.

Safety Class: The safety class addresses the partial safety factor, γ_T, which has the values listed below for each safety class. Unless the pipeline is under construction (LOW safety class) or in a heavily populated or critical area (HIGH safety class), this safety class can be taken as NORMAL.

Safety Class	**LOW (temporary)**	**NORMAL (in-service)**	**HIGH (in-service)**
γ_T	1.7	2.0	2.3

OPTIONAL INPUT VARIABLES:

T : Residual Pipe Tension in kips -- This optional input is used only in the STATIC 3 worksheet when a pipeline passes over an elevated obstruction such as a rock outcropping or man-made object. If the span is induced by a low depression such as a sea floor pit or valley, tension is not considered. It is generally difficult to estimate residual tension in an as-laid pipeline. Unless residual tension is known for certain, it is recommended that this input be left blank and this value will be taken as zero.

S_t : Strouhal Number -- This value is automatically calculated based on distance from the sea floor and overall pipeline diameter including concrete weight coating. This value is normally taken as 0.2. Under certain conditions, however, the automatic calculation may severely diverge from 0.2 or become negative making it necessary to manually input Strouhal Number based on a Reynolds Number criterion. Manually inputting a value here will override the automatic calculation of Strouhal Number. The value entered here must be in the range between 0.15 and 0.45 or an out of range error will result. A graph of Strouhal Number vs. Reynolds Number is included in the main report. Refer to the value for Reynolds Number in the GLOBAL VARIABLES Worksheet.

ζ : Damping Ratio -- The damping ratio is automatically calculated from the logarithmic decrement coefficient that is typically taken as 0.05 for a pipeline free-span. This yields the default damping coefficient of 0.008. This parameter is only used when the 1996 DNV Guidelines are considered. Manually inputting this value will override the automatic calculation of the Damping Ratio. If a more accurate logarithmic decrement coefficient is available, the damping ratio may be manually calculated using the equation below:

$$z = p\left(\frac{\sqrt{\left(\frac{d}{p}\right)^2 + 1} - 1}{d}\right)$$

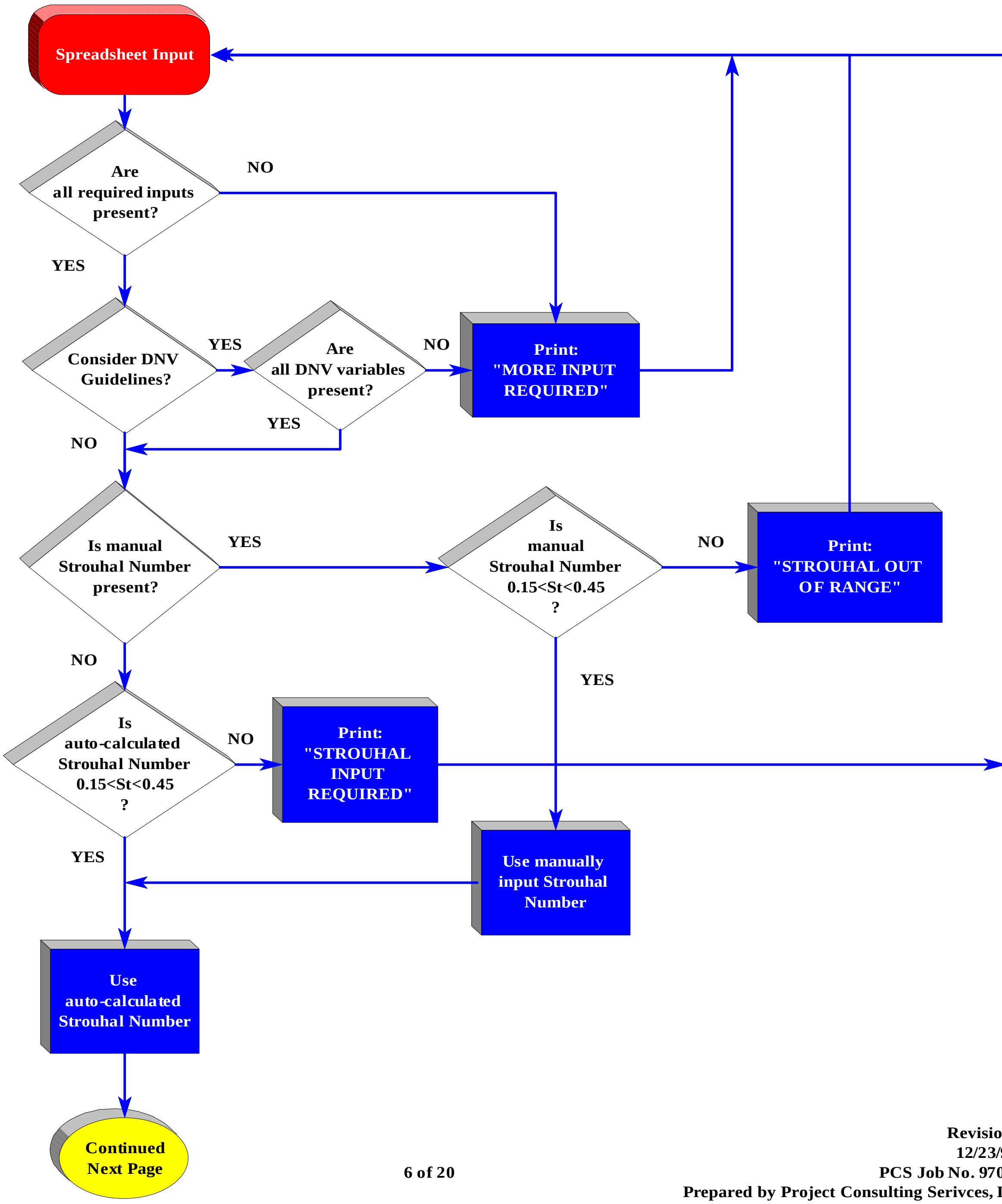
Where:

δ = Logarithmic Decrement Coefficient of Structural Damping

MASTER WORKSHEET FLOW CHART:

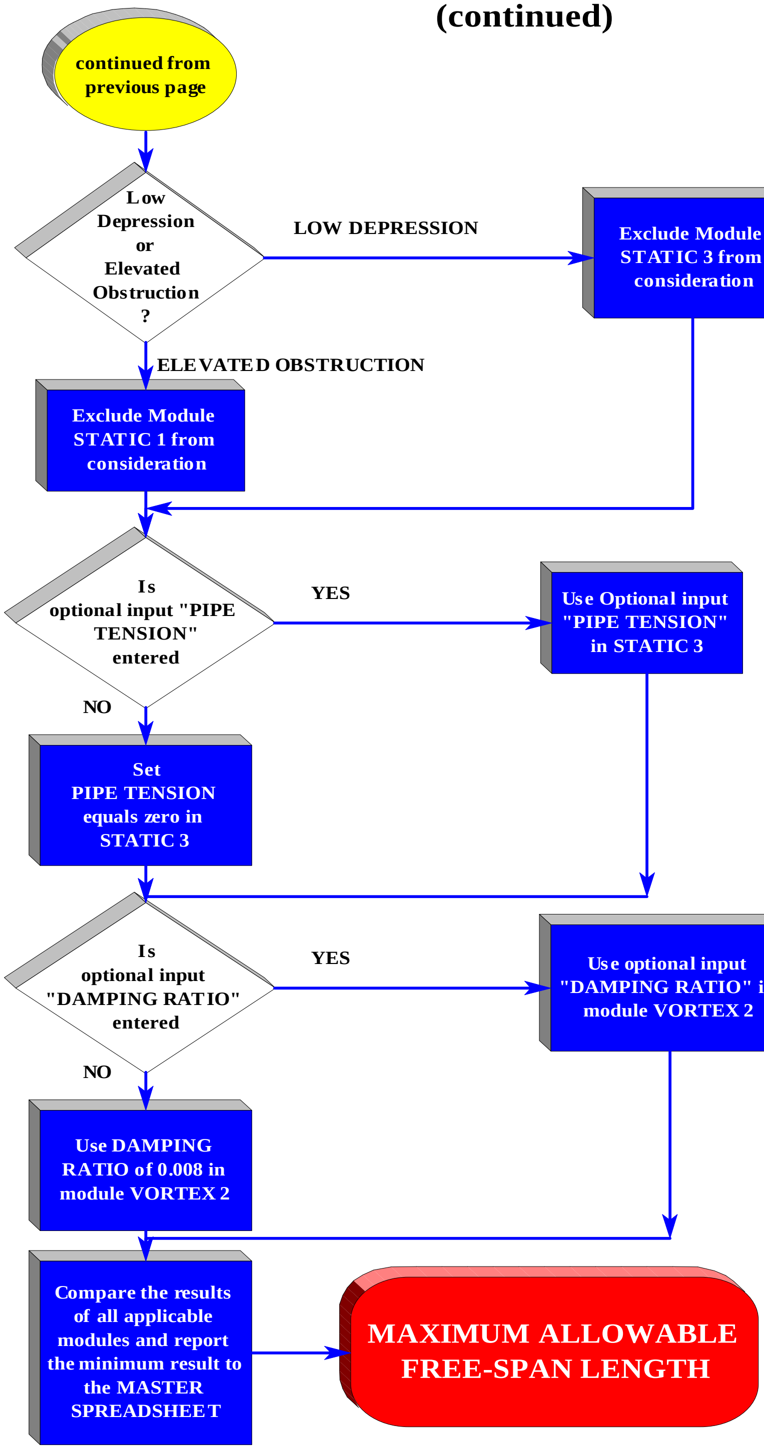
A flow chart describing the input logic of the Master Worksheet is attached below for reference:

Minerals Management Service
Assessment and Analysis of Unsupported Subsea Pipeline Spans
MASTER WORKSHEET FLOW CHART



Revision 0
12/23/97
PCS Job No. 97058
Prepared by Project Consulting Serivces, Inc.

MASTER WORKSHEET FLOW CHART
(continued)



Minerals Management Service
Assessment and Analysis of Unsupported Subsea Pipeline Spans
MASTER WORKSHEET

Refer to REFERENCE GUIDE Worksheet for user instructions

General Information:

☑ Consider 1996 DNV Guidelines developed within the MULTISPAN project

The free span is induced by: ◉ ELEVATED OBSTRUCTION ○ LOW DEPRESSION

Required Input Variables:

D_o=	18.000	in. =	Pipe Outside Diameter
t =	0.562	in. =	Pipe Wall Thickness
E=	2.90E+07	psi =	Young's Modulus
ν_o=	0.300	=	Poisson's Ratio
S_y=	52,000	psi =	Specified Minimum Yield Strength of Pipe
t_c=	1.50	in. =	Concrete Weight Coating Thickness
ρ_{os}=	490.00	$lbs./ft.^3$ =	Density of Steel
ρ_{oc}=	140.00	$lbs./ft.^3$ =	Concrete Weight Coating Density
ρ_{ocn}=	64.00	$lbs./ft.^3$ =	Density of Pipeline Contents
ρ_{ow}=	64.00	$lbs./ft.^3$ =	Density of Sea Water
U=	3.00	ft./s =	Sea Current Velocity for a 100 Yr. Return Period Storm
ν_k=	1.13E-05	$ft.^2/s$ =	Kinematic Viscosity of Sea Water
e=	3.00	ft.=	Gap Between Pipeline and Seafloor
P_{maop}=	1440.00	psi =	Maximum Allowable Operating Pressure
C=	2.55	=	Free-Span Fixity Constant

DNV Guideline-Specific Input Variables:

$C_{m=}$	1.00	=	Added Mass Coefficient for Cross Flow Motion
ψ_R=	1.00	=	Natural Frequency Reduction Factor
ψ_U=	1.00	=	Extreme Current Variability Factor
ψ_D=	1.00	=	Period Transformation Factor
	NORMAL		**Safety Class**

Optional Input Variables:

T=		kips =	Pipe Tension
S_t=		=	Strouhal Number
ζ=		=	Damping Ratio

Final Result:

L=	**107.95**	**feet =**	**Maximum Allowable Free Span Length**

GLOBAL VARIABLES

Description of Calculations:

This worksheet defines the Global Variables for the calculation set. Global variables are typically defined as the variables that repeatedly appear in several of the calculation methods. The unshaded variables are those that are calculated within this worksheet from the inputs of other global variables. GLOBAL VARIABLES derives input from the MASTER Worksheet.

Assumptions:

- Nominal dimensions and properties are assumed unless specified otherwise

Global Variables:

D_o=	18.000	in. =	Pipe Outside Diameter
t =	0.562	in. =	Pipe Wall Thickness
D_i =	16.876	in.=	Pipe Inside Diameter
E=	2.9E+07	psi=	Young's Modulus
ν_o=	0.3	=	Poisson's Ratio
S_y=	52,000	psi=	Specified Minimum Yield Strength of Pipe
t_c=	1.5	in. =	Concrete Weight Coating Thickness
D_{TOT}=	21.0	in.=	Total Diameter of Pipe with Concrete Weight Coating
ρ_{os}=	490	lbs./ft.^{3} =	Density of Steel
ρ_{oc}=	140	lbs./ft.^{3} =	Concrete Weight Coating Density
ρ_{ocn}=	64.0	lbs./ft.^{3} =	Density of Pipeline Contents
ρ_{ow}=	64	lbs./ft.^{3} =	Density of Sea Water
w=	139.6	lbs./ft.	Submerged Weight of Pipe Per Foot
I=	1171.5	in.^{4}	Moment of Inertia of Pipe Cross Section
Z=	130.2	in.^{3}	Pipe Section Modulus
U=	3.0	ft./s	Sea Current Velocity for a 100 Yr. Return Period Storm
ν_k=	1.13E-05	ft.^{2}/s	Kinematic Viscosity of Sea Water[1]
R_E=	464602	=	Reynolds Number
e=	3.00	ft.=	Gap Between Pipeline and Seafloor
S_t=	0.219	=	Strouhal Number[3]
M=	13.90	slugs/ft.=	Dynamic Mass of Submerged Pipe
C=	2.55	=	Beam Fixity Constant[2,4]
f_s=	0.375	Hz=	Strouhal Frequency or Vortex Shedding Frequency

References:

1. Fox, Robert W. and McDonald, Alan T. Introduction to Fluid Mechanics. John Wiley and Sons, Inc. Third Edition. 1985. pg. 682.

2. Shah, B.C., White, C.N., and Rippon, I.J. "Design and Operations Considerations for Unsupported Offshore Pipeline Spans." OTC 5216. Proceedings from the 18th Annual Offshore Technology Conference. Houston, TX. 1986. pg. 5

3. Vitali, L., Mork, K.J., Verley, R., and Malacari, L.E. "The Multispan Project: Response Models for Vortex Induced Vibrations of Submarine Pipelines." Proceedings from OMAE. 1997. pg. 5

4. Nielsen, R., and Gravesen, H., edited by de la Mare, R.F. Advances in Offshore Oil and Gas Pipeline Technology. Gulf Publishing Company. Houston TX, 1985. pg. 326.

STATIC 1: Static Analysis for Low Depression Induced Free Spans

Description of Calculations:

This method is to be used in lieu of the Static 3 method if the pipeline span is induced by a low depression. This method uses the procedure outlined in Offshore Pipeline Design, Analysis, and Methods by A.H. Mouselli[1] to calculate the pipe stress due to low depressions using dimensionless parameters. The graph presented in figure 3.19 for β = 0 is converted to an equation through regression analysis. Once the dimensionless span is determined, the maximum allowable span length is calculated.

Assumptions:

-Thermal Expansion is Negligible
-Pipe Configuration is Geometrically Symmetrical
-Pipe is Flooded with Sea Water
-Axial Pipe Tension Force is 0 lbs.

Global Variables:

D_o=	18.000	in. =	Pipe Outside Diameter
t =	0.562	in. =	Pipe Wall Thickness
D_i =	16.876	in.=	Pipe Inside Diameter
E=	2.9E+07	psi=	Young's Modulus
ν_o=	0.3	=	Poisson's Ratio
S_y=	52,000	psi=	Specified Minimum Yield Strength of Pipe
t_c=	1.5	in. =	Concrete Weight Coating Thickness
D_{TOT}=	21.000	in.=	Total Diameter of Pipe with Concrete Weight Coating
ρ_{os}=	490	lbs./ft.^{3} =	Density of Steel
ρ_{oc}=	140	lbs./ft.^{3} =	Concrete Weight Coating Density
ρ_{ocn}=	64.0	lbs./ft.^{3} =	Density of Pipeline Contents
ρ_{ow}=	64	lbs./ft.^{3} =	Density of Sea Water
I=	1171.5	in.^{4}	Moment of Inertia of Pipe Cross Section
w=	139.6	lbs./ft.=	Submerged Weight of Pipe Per Foot

Local Variables and Calculations:

Determine dimensionless stress by direct substitution. Dimensionless span is determined using equations that are curve fit to Mouselli's Figure 3.19.[1]

σ_m=	41600	=	Maximum Stress Based on Allowables in B31.8 Table A842.22 .[1,2]
c=	9.000	in.=	Pipe Outer Radius
L_c	119.1	ft.=	Characteristic Length
σ_c=	182589.5	psi=	Characteristic Stress
σ_m/σ_c=	0.228	=	Dimensionless Stress
L/L_c	1.866	=	Dimensionless Span

Final Results:

L=	222.25	**=**	**Maximum Allowable Free Span Length**

STATIC 1: Static Analysis for Low Depression Induced Free Spans

References:

1. Mouselli, A.H. Offshore Pipeline Design, Analysis, and Methods, PennWell Publishing Co. 1981. pp. 62-64

2. ASME B31.8-1995 Edition "Gas Transmission and Distribution Piping Systems." The American Society of Mechanical Engineers. 1995. pp. 97-98.

STATIC 2: Free Span Beam Analysis Based on B31.8 Longitudinal and Combined Stress Allowables

Description of Calculations:

This method uses the static longitudinal and combined stress allowables specified in the Offshore Gas Transmission Section of ASME B31.8. Beam flexural formulas are used to back out a maximum span length based on these code allowables.

Assumptions:

-Thermal Expansion is Negligible
-Pipe is Fully Restrained at Each End
-End Cap Effect is not Considered
-Tangential Shear Stress, S_s=O
-Pipe is Flooded with Seawater
-Axial Pipe Tension Force is 0 lbs.

Global Variables:

D_o=	18.000	in. =	Pipe Outside Diameter
t =	0.562	in. =	Pipe Wall Thickness
D_i =	16.876	in.=	Pipe Inside Diameter
E=	2.9E+07	psi=	Young's Modulus
ν_o=	0.3	=	Poisson's Ratio
S_y=	52,000	psi=	Specified Minimum Yield Strength of Pipe
t_c=	1.5	in. =	Concrete Weight Coating Thickness
D_{TOT}=	21.000	in.=	Total Diameter of Pipe with Concrete Weight Coating
ρ_{os}=	490	$lbs./ft.^3$ =	Density of Steel
ρ_{oc}=	140	$lbs./ft.^3$ =	Concrete Weight Coating Density
ρ_{ocn}=	64.0	$lbs./ft.^3$ =	Density of Pipeline Contents
ρ_{ow}=	64	$lbs./ft.^3$ =	Density of Seawater
Z=	130.17	$in.^3$=	Pipe Section Modulus
w=	139.6	lbs./ft.=	Submerged Weight of Pipe Per Foot

Local Variables & Calculations:

Determine available bending stress based on longitudinal stress allowables in B31.8 Section A842.222 with Poisson's Effect considered:[1]

σ_{Lmax}=	41,600	psi=	Longitudinal Stress Limit[1]
P_{maop}=	1,440	psi =	Maximum Allowable Operating Pressure
σ_H=	23,060	psi=	Hoop Stress Limit
σ_p=	-6,918	psi=	Poisson's Effect
σ_{b1}=	34,682	psi=	Available Bending Stress Based on Longitudinal Stress Allowables[1]

STATIC 2: Free Span Beam Analysis Based on B31.8 Longitudinal and Combined Stress Allowables

Local Variables & Calculations (continued)

Determine available bending stress based on longitudinal stress allowables in B31.8 Section A842.223 with Poisson's Effect considered:

σ_{Cmax}=	46,800	psi =	Combined Stress Limit[1]
σ_{L1}=	53,855	psi =	Longitudinal Stress Component 1
σ_{L2}=	-30,795	psi =	Longitudinal Stress Component 2
σ_{b2}=	23,877	psi =	Available Bending Stress Based on Von Mises Combined Stress Limits[1] including Poisson's Effect.

Determine Maximum Span Using Classical Flexural Beam Relations:

σ_b=	23,877	psi =	Maximum Available Bending Stress

Final Results:

L=	**136.22**	**ft=**	**Minimum Allowable Free Span Length[2]**

References:

1. ASME B31.8-1995 Edition "Gas Transmission and Distribution Piping Systems." The American Society of Mechanical Engineers. 1995. pp. 97-98.

2. Avallone, E.A. and Baumeister, T., III. Marks' Standard Handbook for Mechanical Engineers. Ninth Edition. McGraw Hill Book Company. 1987. pg. 5-24.

3. Shah, B.C., White, C.N. and Rippon, I.J. "Design and Operational Considerations for Unsupported Offshore Pipeline Spans." O.T.C. 5216. Proceedings from the 18th Annual Offshore Technology Conference. Houston, TX. 1986. pg. 5.

STATIC 3: Static Analysis for Elevated Obstruction Induced Free Spans

Description of Calculations:

This method is to be used in lieu of STATIC 1 if the pipeline span is induced by an elevated obstruction. This method develops a procedure, based on Offshore Pipeline Design, Analysis, and Methods by A. H. Mouselli, Section 3.82, that determines the maximum allowable pipeline span length induced by an elevated obstruction. Non-dimensional parameters are first determined. The graphs presented in Figures 3.24 and 3.25 are converted to equations through regression curve fitting. After dimensionless spans are found for β=O and β=10, the dimensionless span is found for β_{actual} through linear interpolation. The maximum span length can then be calculated from the dimensionless span.

Assumptions:

-Thermal Expansion Effects are Negligible
-Pipe Configuration is Geometrically Symmetrical
-Pipe is Flooded with Seawater

Global Variables:

D_o=	18.000	in. =	Pipe Outside Diameter
t =	0.562	in. =	Pipe Wall Thickness
D_i =	16.876	in.=	Pipe Inside Diameter
E=	2.9E+07	psi=	Young's Modulus
σ_o=	0.3	=	Poisson's Ratio
S_y=	52,000	psi=	Specified Minimum Yield Strength of Pipe
t_c=	1.5	in. =	Concrete Weight Coating Thickness
D_{TOT}=	21.000	in.=	Total Diameter of Pipe with Concrete Weight Coating
ρ_{os}=	490	lbs./ft.^{3} =	Density of Steel
ρ_{oc}=	140	lbs./ft.^{3} =	Concrete Weight Coating Density
ρ_{ocn}=	64.0	lbs./ft.^{3} =	Density of Pipeline Contents
ρ_{ow}=	64	lbs./ft.^{3} =	Density of Sea Water
I=	1171.5	in.^{4}=	Moment of Inertia of Pipe Cross Section
w=	139.6	lbs./ft.	Submerged Weight of Pipe per Foot

Local Variables & Calculations:

Dimensionless parameters are determined. Direct substitution is used to compute dimensionless stress and dimensionless tension. Dimensionless elevation and dimensionless span are determined using equations that are curve fit to Mouselli's Figure 3.25 and Figure 3.24, respectively. Actual dimensionless span is determined by linear interpolation between β=0 and β=10.[1]

σ_m=	41600	psi=	Maximum Stress Based on Allowables in Table A842.22[2]
c=	9	in=	Pipe Outer Radius
L_c=	119.1	ft.	Characteristic Length
σ_c=	182589.5	psi=	Characteristic Stress
T=	0	kips=	Pipe Tension
β=	0.000	=	Dimensionless Tension

STATIC 3: Static Analysis for Elevated Obstruction Induced Free Spans

Local Variables & Calculations (continued):

σ_m/σ_c=	0.228	=	Dimensionless Stress
$100\delta/L_c$=	2.827	=	Maximum Allowable Dimensionless Elevation due to Elevated Obstruction
$L/L_c\|_{\beta=0}$=	2.356	=	Dimensionless Span at β=0
$L/L_c\|_{\beta=10}$=	2.698	=	Dimensionless Span at β=10
$L/L_c\|_{\beta actual}$=	2.356	=	Dimensionless Span at β_{actual}

Final Results:

L= 280.7 ft.= Maximum Allowable Free Span Length

References:

1. Mouselli, A.H. Offshore Pipeline Design, Analysis, and Methods, PennWell Publishing Co., 1981. pp. 61-64.

2. ASME B31.8-1995 Edition "Gas Transmission and Distribution Piping Systems." The American Society of Mechanical Engineers. 1995. pp. 97-98.

VORTEX 1: General Vortex Induced Vibration (VIV) Analysis

Description of Calculations:

This method uses the vortex shedding frequency and the natural frequency of an unsupported pipeline span to determine the maximum allowable length. Vortex-induced oscillation relations are based on those developed by Nielsen and Gravesen[1] and Mouselli[3].

Assumptions:

- In-line Vortex Induced Vibrations not Considered
- Pipe is Flooded with Sea Water
- Pipe is Partially Fixed at Each End[2]
- Flow Incident Angle on Pipe is 90°

Global Variables:

D_o=	18.000	in. =	Pipe Outside Diameter
t =	0.562	in. =	Pipe Wall Thickness
D_i =	16.876	in.=	Pipe Inside Diameter
E=	2.9E+07	psi=	Young's Modulus
ν_o=	0.3	=	Poisson's Ratio
S_y=	52,000	psi=	Specified Minimum Yield Strength of Pipe
t_c=	1.5	in. =	Concrete Weight Coating Thickness
D_{TOT}=	21.000	in.=	Total Diameter of Pipe with Concrete Weight Coating
ρ_{os}=	490	lbs./ft.^{3} =	Density of Steel
ρ_{oc}=	140	lbs./ft.^{3} =	Concrete Weight Coating Density
ρ_{ocn}=	64.0	lbs./ft.^{3} =	Density of Pipeline Contents
ρ_{ow}=	64	lbs./ft.^{3} =	Density of Sea Water
M=	13.90	slugs =	Dynamic Mass of Submerged Pipe
I=	1171.5	in.^{4}=	Moment of Inertia of Pipe Cross Section
C=	2.55	=	Free Span Fixity Constant
S_t=	0.219	=	Strouhal Number
e=	3.00	ft. =	Gap Between Seafloor and Pipeline
f_s=	0.375	Hz=	Strouhal Frequency or Vortex Shedding Frequency

Local Variables & Calculations

Maximum allowable span length is calculated by comparing the vortex shedding frequency to the natural frequency of a given span and solving for L.

L=	**140.10**	**ft.**	**Maximum Allowable Free Span Length**

VORTEX 1: General Vortex Induced Vibration (VIV) Analysis

References:

1. Neilsen, R., Gravesen, H., edited by delaMare, R.F. Advances in Offshore Oil and Gas Pipeline Technology. Gulf Publishing Company. Houston, TX. 1985. pg. 326.

2. Shah, B.C., White, C.N., and Rippon, I.J. "Design and Operational Considerations for Unsupported Offshore Pipeline Spans." OTC 5216. Proceedings from the 18th Annual Offshore Technology Conference. 1986. pg. 5.

3. Mouselli, A.H. Offshore Pipeline Design, Analysis, and Methods. PennWell Publishing Co. 1981. pp. 50-52.

VORTEX 2: Cross Flow VIV Analysis

Description of Calculations:

This analysis method is based on the research from the MULTISPAN project. The method calculates the maximum allowable free span length by preventing the onset of cross flow vortex induced vibrations (VIV). Natural frequency of the span is determined using the method outlined in Nielsen and Gravesen[7].

Assumptions:

-Axial Pipe Tension Equals 0 lbs.
-Pipe is Flooded with Sea Water
-Flow Incidence Angle 90° with Pipe
-Turbulence Less than 8%; No Extreme Current Variability
-Infinitely Long Cylinder

Global Variables:

D_o=	18.000	in. =	Pipe Outside Diameter
t =	0.562	in. =	Pipe Wall Thickness
D_i =	16.876	in.=	Pipe Inside Diameter
E=	2.9E+07	psi=	Young's Modulus
ν_o=	0.3	=	Poisson's Ratio
S_y=	52,000	psi=	Specified Minimum Yield Strength of Pipe
t_c=	1.5	in. =	Concrete Weight Coating Thickness
D_{TOT}=	21.000	in.=	Total Diameter of Pipe with Concrete Weight Coating
ρ_{os}=	490	$lbs./ft.^3$ =	Density of Steel
ρ_{oc}=	140	$lbs./ft.^3$ =	Concrete Weight Coating Density
ρ_{ocn}=	64.0	$lbs./ft.^3$ =	Density of Pipeline Contents
ρ_{ow}=	64	$lbs./ft.^3$ =	Density of Sea Water
I=	1171.5	$in.^4$	Moment of Inertia of Pipe Cross Section
M=	13.90	=	Dynamic Mass of Submerged Pipe
C=	2.55	=	Free Span Fixity Constant
U=	3.0	=	Sea Current Velocity for a 100 Year Storm

Local Variables & Calculations:

Section 1

The damping ratio is determined by solving the logarithmic decrement equation for ζ[4]. The recommended value for damping ratio was determined by using the value of 0.05 for the logarithmic decrement[1].

ζ=	0.0080	=	Damping ratio

VORTEX 2: Cross Flow VIV Analysis

Section 2:

The stability parameter is a function of specific weight, added mass coefficient[2], and Strouhal number[8].

Calculations:

w_s=	194.10	=	Dry Weight of Pipe Plus Weight Coating
w_w=	153.94	=	Displaced Weight of Water
ρ_s/ρ	1.261	=	Specific Gravity of Pipeline
$C_{m=}$	1.00	=	Added Mass Coefficient for Cross Flow Motion
S_t=	0.219	=	Strouhal Number
K_s=	0.020	=	Stability Parameter

Section 3:

The reduced velocity is a function of pipe diameter and sea current velocity. 100 yr. Return Period Storm data is used to determine a reference velocity. Partial safety factors are based on the referenced sources.

Calculations:

γ_T=	2.0	=	Safety Class Partial Coefficient, Assume Normal In-Service Pipeline[3]
ψ_D=	1	=	Period Transformation Factor[6]
ψ_R=	1	=	Natural Frequency Reduction Factor[3]
ψ_u=	1	=	Extreme Current Variability Factor[5]
U=	3.0	=	Sea Current for a 100 Year Return Period Storm
$V_{r,onset}$=	3.802	=	Reduced Velocity

Section 4:

This section determines the maximum allowable span length based by equating the free span natural frequency to the partial safety factor equation given in Section 3.

Calculation:

f_n	0.90	=	Natural Pipeline Frequency Given by Mouselli, 1981.[4]

Final Results:

L=	**107.95**	=	**Maximum Allowable Free Span Length**

VORTEX 2: Cross Flow VIV Analysis

References:

1. Tsahalis, D.T. and Jones, W.T. "Vortex-Induced Vibrations of a Flexible Cylinder Near a Plane Boundary in Steady Flow." OTC 1231. Proceedings from the 14th Annual Offshore Technology Conference. Houston, TX. 1982. pg. 1.

2. DNVC CN30.5. "Environmental Conditions and Environmental Loads." Det Norske Veritas, Norway. 1991. pg. 23.

3. Mork, K.J. and Vitali, L. "An Approach to Design Against Cross-Flow VIV for Submarine Pipelines." Dynamics of Structures. Aalborg University, Denmark. 1996. pg. 5.

4. Rao, S.S. Mechanical Vibrations. Addison-Wesley Publishing Company, Inc. Second Edition. 1990. pg. 89.

5. Mork, K.J., Vitali, L., and Verly, R. "The MULTISPAN Project: Design Guideline for Free Spanning Pipelines." Proceedings from the 16th International Conference on Offshore Mechanics and Arctic Engineering. American Society of Mechanical Engineers. Yokahama, Japan. 1997. pg. 3.

6. Mathiesen, M., Hansen, E.A., Andersen, O.J., and Bruschi, R. "The MULTISPAN Project: Near Seabed Flow in Macroroughness Areas." Proceedings from the 16th Annual International Conference on Offshore Mechanics and Arctic Engineering." American Society of Mechanical Engineers. Yokahama, Japan. 1997. pg. 20.

7. Neilsen, R. and Gravesen, H., edited by delaMare, R.F. Advances in Offshore Oil and Gas Pipeline Technology. Gulf Publishing Company. Houston, TX. 1985. pg. 326.

8. Vitali, L., Mork, K.J., Verley, R., and Malacari, L.E. "The MULTISPAN Project: Response Models for Vortex Induced Vibrations of Submarine Pipelines." Proceedings from the 16th International Conference on Offshore Mechanics and Arctic Engineering. American Society of Mechanical Engineers. Yokohama, Japan. pg. 5.

APPENDIX C:

Example Hand Calculations

RESULT SUMMARY (1 OF 2)

REQUIRED INPUT:

- CONSIDERATION IS GIVEN FOR 1996 DNV GUIDELINES DEVELOPED WITHIN THE MULTISPAN PROJECT.
- FREE SPAN IS INDUCED BY ELEVATED OBSTRUCTION

$D_o = 18.00$ IN = PIPE OUTSIDE DIAMETER

$t = 0.562$ IN = PIPE WALL THICKNESS

$E = 2.9 \times 10^7$ PSI = YOUNG'S MODULUS

$\nu_o = 0.300$ = POISSON'S RATIO

$S_y = 52{,}000$ PSI = SPECIFIED MINIMUM YIELD STRENGTH OF PIPE

$t_c = 1.50$ IN = CONCRETE WEIGHT COATING THICKNESS

$\rho_{os} = 490$ LBS/FT^3 = DENSITY OF STEEL

$\rho_{oc} = 140$ LBS/FT^3 = CONCRETE WEIGHT COATING DENSITY

$\rho_{ocn} = 64$ LBS/FT^3 = DENSITY OF PIPELINE CONTENTS

$\rho_{ow} = 64$ LBS/FT^3 = DENSITY OF SEAWATER

$U = 3.0$ FT/S = SEA CURRENT VELOCITY FOR A 100YR RETURN PERIOD STORM

$\nu_K = 1.13 \times 10^{-5}$ FT^2/S = KINEMATIC VISCOSITY OF SEA WATER

$e = 3.0$ FT = GAP BETWEEN PIPELINE AND SEA FLOOR

$P_{maop} = 1{,}440$ PSI = MAXIMUM ALLOWABLE OPERATING PRESSURE

$C = 2.55$ = FREE SPAN FIXITY CONSTANT

$C_M = 1.00$ = ADDED MASS COEFFICIENT FOR CROSS FLOW MOTION

$\psi_R = 1.00$ = NATURAL FREQUENCY REDUCTION FACTOR

$\psi_u = 1.00$ = EXTREAM CURRENT VARIABILITY FACTOR

$\psi_D = 1.00$ = PERIOD TRANSFORMATION FACTOR

$\gamma_T = 2.00$ = NORMAL SAFETY CLASS COEFFICIENT

RESULT SUMMARY (2 OF 2)

OUTPUT:

STATIC 1: STATIC ANALYSIS FOR LOW DEPRESSION INDUCED FREE SPANS

MAFSL : 222 FT.

STATIC 2: FREE SPAN BEAM ANALYSIS BASED ON B31.8 LONGITUDINAL AND COMBINED STRESS ALLOWABLES

MAFSL : 136. FT.

STATIC 3: STATIC ANALYSIS FOR ELEVATED OBSTRUCTION INDUCED FREE SPANS

MAFSL : 281 FT.

VORTEX 1: GENERAL VORTEX INDUCED VIBRATION (VIV) ANALYSIS

MAFSL : 140 FT.

VORTEX 2: CROSS FLOW VIV ANALYSIS

MAFSL : 108 FT.

RESULT INTERPRETATION:

METHODS CONSIDERED ARE: STATIC 2, STATIC 3, VORTEX 1, AND VORTEX 2. STATIC 1 IS NOT CONSIDERED AS SPAN IS INDUCED BY ELEVATED OBSTRUCTION

MINIMUM STATIC MAFSL : 136 FT. (STATIC 2)

MINIMUM DYNAMIC MAFSL : 108 FT. (VORTEX 2)

COMBINED ANALYSIS METHOD GOVERNING MAFSL: 108 FT. (VORTEX 2)

CAM GOVERNING MAFSL SHOULD THEN BE COMPARED TO OTHER SUPPORTING DATA AND PIPELINE REQUIREMENTS TO EVALUATE IF THESE THEORETICAL RESULTS REFLECT ACTUAL PIPELINE OPERATING CONDITIONS.

GLOBAL VARIABLES (1 OF 3)

DESCRIPTION OF CALCULATIONS:

THIS WORKSHEET DEFINES THE GLOBAL VARIABLES FOR THE CALCULATION SET. GLOBAL VARIABLES ARE TYPICALLY DEFINED AS THE VARIABLES THAT REPEATEDLY APPEAR ON SEVERAL CALCULATION WORKSHEETS.

ASSUMPTIONS:

NOMINAL DIMENSIONS AND PROPERTIES ARE ASSUMED UNLESS SPECIFIED OTHERWISE

GLOBAL VARIABLES:

$D_o = 18.00$ IN. = PIPE OUTSIDE DIAMETER

$t = 0.562$ IN. = PIPE WALL THICKNESS

$D_i = D_o - 2t = (18.00 \text{ IN.}) - 2(0.562 \text{ IN.}) = 16.876$ IN. = PIPE INSIDE DIAMETER

$E = 2.9 \times 10^7$ psi = YOUNG'S MODULUS

$\nu_o = 0.30$ = POISSON'S RATIO

$S_y = 52{,}000$ psi = SPECIFIED MINIMUM YIELD STRENGTH OF PIPE

$t_c = 1.50$ IN. = CONCRETE WEIGHT COATING THICKNESS

$D_{TOT} = D_o + 2t_c = (18.00 \text{ IN.}) + 2(1.50 \text{ IN.}) = 21.00$ IN.
= TOTAL DIAMETER OF PIPE WITH CONCRETE COATING

$\rho_{os} = 490$ LBS./FT.^{3} = STEEL DENSITY

$\rho_{oc} = 140$ LBS/FT.^{3} = CONCRETE WEIGHT COATING DENSITY

$\rho_{ocn} = 64$ LB./FT.^{3} = DENSITY OF PIPELINE CONTENTS

$\rho_{ow} = 64$ LBS./FT.^{3} = SEAWATER DENSITY

$$W = \frac{\pi}{4}\left[(D_o^2 - D_i^2)(\rho_{os}) + (D_{TOT}^2 - D_o^2)(\rho_{oc}) + (D_i^2)(\rho_{ocn}) - (D_{TOT}^2)(\rho_{ow})\right]$$

$$= \frac{\pi}{4}\left[\left((18.00 \text{ IN})^2 - (16.876 \text{ IN.})^2\right)(490 \text{ LBS./FT.}^3) + \left((21.00 \text{ IN.})^2 - (18.00 \text{ IN})^2\right)(140 \text{ LBS/FT.}^3) + (16.876 \text{ IN.})^2 (64 \text{ LB/FT.}^3) - (21.000 \text{ IN})^2 (64 \text{ LB./FT.}^3)\right] \Big/ \left(\frac{1 \text{ FT}^2}{144 \text{ IN}^2}\right)$$

$= 139.6$ LB./FT = SUBMERGED WEIGHT OF PIPE PER FOOT

GLOBAL VARIABLES (CONTINUED) (2 OF 3)

GLOBAL VARIABLES (CONTINUED):

$$I = \frac{\pi}{64}\left(D_o^4 - D_i^4\right) = \frac{\pi}{64}\left((18.00\text{ in.})^4 - (16.876\text{ in.})^4\right) = 1,171.5\text{ in}^4$$ = PIPE MOMENT OF INERTIA

$$Z = \frac{\pi}{32}\left(\frac{D_o^4 - D_i^4}{D_o}\right) = \frac{\pi}{32}\left(\frac{(18.00\text{ in.})^4 - (16.876\text{ in.})^4}{18.00\text{ in.}}\right) = 130.17\text{ in}^3$$ = PIPE SECTION MODULUS

$U = 3.0$ FT./S = SEA CURRENT VELOCITY FOR A 100 YR. RETURN PERIOD STORM

$\nu_K = 1.13 \times 10^{-5}$ FT.²/S = KINEMATIC VISCOSITY OF SEAWATER AT 70°F (20°C)[1]

$$R_E = \frac{U D_{TOT}}{\nu_K} = \frac{(3.0\text{ FT/S})(21.00\text{ IN})}{(1.13\times 10^{-5}\text{ FT}^2/\text{S})} = 4.65\times 10^5$$ = REYNOLDS NUMBER

$e = 3$ FT. = GAP BETWEEN PIPELINE AND SEAFLOOR

$$S_t = 0.27 - 0.03\left(\frac{e}{D_{TOT}}\right) = 0.27 - 0.03\left(\frac{3\text{ FT}}{21\text{ IN}}\right)\left(\frac{12\text{ IN}}{1\text{ FT}}\right) = 0.219$$ = STROUHAL NUMBER[3]

$$M = \frac{\pi}{4}\left[\left(D_o^2 - D_i^2\right)(\rho_{os}) + \left(D_{TOT}^2 - D_o^2\right)(\rho_{oc}) + (D_i^2)(\rho_{ocn}) + (D_{TOT}^2)(\rho_{ow})\right]$$

$$= \frac{\pi}{4}\left[\left((18.00\text{ in})^2 - (16.876\text{ in})^2\right)(490\text{ LB/FT}^3) + \left((21.00\text{ in})^2 - (18.00\text{ in})^2\right)(140\text{ LB/FT}^3) + (16.876\text{ in.})^2(64\text{ LB/FT}^3) + \left((21.00\text{ in})^2(64\text{ LB/FT}^3)\right)\right]\left(\frac{1\text{ FT}^2}{144\text{ in}^2}\right)\left(\frac{1\text{ slug}}{32.2\text{ LBS}}\right)$$

$= 13.90\ \frac{\text{SLUGS}}{\text{FT.}}$ = DYNAMIC MASS OF SUBMERGED PIPE

$$C = \frac{K\pi}{2} = \frac{\left((1.00 + 2.25)/2\right)\pi}{2} = 2.55$$

K = 1.00 FOR SIMPLY SUPPORTED PIPE ENDS; 2.25 FOR FULLY FIXED PIPE ENDS; AVERAGE IS ASSUME TO APPROXIMATE ACTUAL CONDITIONS.[2,4]

$$f_s = \frac{S_t U}{D_{TOT}} = \frac{(0.219)(3.0\text{ FT./S})}{(21.00\text{ in.})}\left(\frac{12\text{ in.}}{1\text{ FT.}}\right) = 0.375\text{ Hz}$$ = VORTEX SHEDDING FREQUENCY

GLOBAL VARIABLES (CONTINUED) (3 OF 3)

REFERENCES:

1.) FOX, ROBERT W. AND McDONALD, ALAN T. INTRODUCTION TO FLUID MECHANICS. JOHN WILEY AND SONS, INC. THIRD EDITION. 1985. PG. 682.

2.) SHAH, B. C., WHITE, C. N., AND RIPPON, I. J. "DESIGN AND OPERATIONAL CONSIDERATIONS FOR THE UNSUPPORTED OFFSHORE PIPELINE SPANS." O.T.C. 5216. PROCEEDINGS FROM THE 18TH ANNUAL OFFSHORE TECHNOLOGY CONFERENCE. HOUSTON, TX. 1986. PG. 5.

3.) VITALI, L., MORK, K. J., VERLEY, R., AND MALACARI, L. E. "THE MULTISPAN PROJECT: RESPONCE MODELS FOR VORTEX INDUCED VIBRATION OF SUBMARINE PIPELINES." PROCEEDINGS FROM THE 16TH INTERNATIONAL CONFERENCE ON OFFSHORE MECHANICS AND ARCTIC ENGINEERING. AMERICAN SOCIETY OF MECHANICAL ENGINEERS. YOKAHAMA, JAPAN. PG. 5.

4.) NIELSEN, R. AND GRAVESEN, H., EDITED BY DE LA MARE, R.F. ADVANCES IN OFFSHORE OIL AND GAS PIPELINE TECHNOLOGY. GULF PUBLISHING COMPANY. HOUSTON, TX. 1985. PG. 326.

STATIC 1: STATIC ANALYSIS FOR LOW DEPRESSION INDUCED FREE SPANS (1 OF 2)

DESCRIPTION OF CALCULATIONS:

THIS METHOD IS USED IN LIEU OF THE STATIC 3 METHOD IF THE PIPELINE SPAN IS INDUCED BY A LOW DEPRESSION. THIS METHOD USES THE PROCEDURE OUTLINED BY MOUSELLI[1] TO CALCULATE THE PIPE STRESS DUE TO LOW DEPRESSIONS USING DIMENSIONLESS PARAMETERS. THE GRAPH PRESENTED IN FIG. 3.19 FOR $\beta=0$ IS CONVERTED TO AN EQUATION BY REGRESSION CURVE FITTING. ONCE THE DIMENSIONLESS SPAN IS DETERMINED, THE MAXIMUM ALLOWABLE SPAN LENGTH IS CALCULATED.

ASSUMPTIONS:

THERMAL EXPANSION IS NEGLIGIBLE
PIPE CONFIGURATION IS GEOMETRICALLY SYMMETRICAL
PIPE IS FLOODED WITH SEAWATER
AXIAL PIPE TENSION FORCE IS 0 LBS.

GLOBAL VARIABLES:

$D_o = 18.00$ IN = PIPE OUTSIDE DIAMETER

$I = 1171.5$ IN^4 = MOMENT OF INERTIA OF PIPE CROSS SECTION

$E = 2.9 \times 10^7$ psi = YOUNG'S MODULUS

$w = 139.6$ LBS/FT = SUBMERGED WEIGHT OF PIPE

$S_y = 52{,}000$ psi = SPECIFIED MINIMUM YIELD STRENGTH OF PIPE

LOCAL VARIABLES:

DETERMINE MAXIMUM SPAN LENGTH BASED ON B31.8 ALLOWABLES:

$\sigma_{max} = (0.80) S_y = 41{,}600$ psi = MAXIMUM ALLOWABLE STRESS PER B31.8 TABLE A842.22[2]

$c = D_o/2 = (18.00)/2 = 9.00$ IN = PIPE OUTER RADIUS

$$L_c = \left[\left(EI/144\tfrac{in^2}{ft^2}\right)/w\right]^{1/3} = \left[\left(2.9\times10^7\ psi\right)\left(1171.5\ in^4 \Big/ 144\tfrac{in^2}{ft^2}\right)\Big/ 139.6\ ^{LBS}/_{FT}\right]^{1/3}$$

$= 119.1$ FT CHARACTERISTIC LENGTH

STATIC 1: STATIC ANALYSIS FOR LOW DEPRESSION INDUCED FREE SPANS (CONTINUED) (2 OF 2)

LOCAL VARIABLES (CONTINUED):

$$\sigma_c = \frac{Ec}{L_c} = \frac{(2.9\times10^7 \text{ psi})(9.0 \text{ in})}{(119.1 \text{ FT})(12 \text{ in/FT})} = 182{,}619.6 \text{ PSI} = \text{CHARACTERISTIC STRESS}$$

$$\beta = \frac{T}{w L_c} = \frac{0 \text{ LBS}}{(139.6 \text{ LB/FT})(119.1 \text{ FT})} = 0 = \text{DIMENSIONLESS TENSION}$$

$$\frac{\sigma_m}{\sigma_c} = \frac{(41{,}600 \text{ psi})}{(182{,}619.6 \text{ psi})} = 0.228 = \text{DIMENSIONLESS STRESS}$$

$$\left.\frac{L}{L_c}\right|_{\beta=0} = 0.1120 + 10.98\left(\frac{\sigma_m}{\sigma_c}\right) - 16.71\left(\frac{\sigma_m}{\sigma_c}\right)^2 + 10.11\left(\frac{\sigma_m}{\sigma_c}\right)^3, \quad 0 \le \frac{\sigma_m}{\sigma_c} \le 0.835$$

$$= 0.1120 + 10.98(0.228) - 16.71(0.228)^2 + 10.11(0.228)^3$$

$$= 1.867 = \text{DIMENSIONLESS SPAN}$$

$$L = \left(\frac{L}{L_c}\right)(L_c) = (1.867)(119.1 \text{ FT}) = 222.3 \text{ FT} = \text{MAXIMUM ALLOWABLE FREE SPAN LENGTH}$$

REFERENCES:

1.) MOUSELLI, A.H. OFFSHORE PIPELINE DESIGN, ANALYSIS, AND METHODS. PENNWELL PUBLISHING CO. 1981. pp 61-63.

2.) ASME B31.8-1995 EDITION, "GAS TRANSMISSION AND DISTRIBUTION PIPING SYSTEMS." AMERICAN SOCIETY OF MECHANICAL ENGINEERS, 1995. pp. 97-98.

STATIC 2: FREE SPAN BEAM ANALYSIS BASED ON B31.8 LONGITUDINAL AND COMBINED STRESS ALLOWABLES (1 OF 3)

DESCRIPTION OF CALCULATIONS:

THIS METHOD USES THE STATIC LONGITUDINAL AND COMBINED STRESS ALLOWABLES SPECIFIED IN THE OFFSHORE GAS TRANSMISSION SECTION OF ASME B31.8[1]. BEAM FLEXURAL FORMULAS ARE USED TO EVALUATE THE MAXIMUM ALLOWABLE SPAN LENGTH BASED ON THESE CODE ALLOWABLES.

ASSUMPTIONS:

THERMAL EXPANSION IS NEGLIGIBLE

PIPE IS FULLY RESTRAINED AT EACH END

END CAP EFFECT IS NOT CONSIDERED

TANGENTIAL SHEAR STRESS, $S_s = 0$

PIPE IS FLOODED WITH SEAWATER

AXIAL PIPE TENSION IS 0 LBS

GLOBAL VARIABLES:

$D_o = 18.00$ IN = PIPE OUTSIDE DIAMETER

$t = 0.562$ IN = PIPE WALL THICKNESS

$t_c = 1.50$ IN = CONCRETE WEIGHT COATING THICKNESS

$\rho_{oc} = 140$ LBS/FT³ = CONCRETE WEIGHT COATING DENSITY

$\rho_{os} = 490$ LBS/FT³ = DENSITY OF STEEL

$\rho_{ocn} = 64$ LBS/FT³ = DENSITY OF PIPELINE CONTENTS

$w = 139.6$ LBS/FT. = SUBMERGED WEIGHT OF PIPE PER FOOT

$Z = 130.17$ IN³ = PIPE SECTION MODULUS

$S_y = 52,000$ PSI = SPECIFIED MINIMUM YIELD STRENGTH

$\rho_{ow} = 64$ LBS/FT³ = DENSITY OF SEAWATER

$D_i = 16.876$ IN = PIPE INSIDE DIAMETER

$\nu_o = 0.30$ = POISSON'S RATIO

STATIC 2: FREE SPAN BEAM ANALYSIS BASED ON B31.8 LONGITUDINAL AND COMBINED STRESS ALLOWABLES (CONTINUED) (2 OF 3)

LOCAL VARIABLES:

DETERMINE AVAILABLE BENDING STRESS BASED ON LONGITUDINAL STRESS ALLOWABLES IN B31.8 SECTION A842.222 WITH POISSON'S EFFECT CONSIDERED:

$$\sigma_{L_{MAX}} = (0.8) S_y = (0.8)(52{,}000\ \text{psi}) = 41{,}600\ \text{psi} = \text{LONGITUDINAL STRESS LIMIT}$$

$$P_{MAOP} = 1440\ \text{psi} = \text{MAXIMUM ALLOWABLE OPERATING PRESSURE}$$

$$\sigma_H = \frac{P_{MAOP} D_o}{2t} = \frac{(1{,}440\ \text{psi})(18.00\ \text{in})}{2(0.562\ \text{in})} = 23{,}060\ \text{psi} = \text{HOOP STRESS LIMIT}$$

$$\sigma_P = -\nu_o \sigma_H = -(0.30)(23{,}060\ \text{psi}) = -6{,}918\ \text{psi} = \text{POISSON'S EFFECT}$$

$$\sigma_{b1} = \left\{ \text{THE LESSER OF} \begin{array}{l} \left|\sigma_{L_{MAX}} - \sigma_P\right| = \left|(41{,}600\ \text{psi}) - (-6{,}918\ \text{psi})\right| = \left|48{,}518\ \text{psi}\right| \\ \left|-\sigma_{L_{MAX}} - \sigma_P\right| = \left|(-41{,}600\ \text{psi}) - (-6{,}918\ \text{psi})\right| = \left|-34{,}682\ \text{psi}\right| \end{array} \right\}$$

$= 34{,}682$ psi $=$ AVAILABLE BENDING STRESS BASED ON LONGITUDINAL STRESS ALLOWABLES

DETERMINE AVAILABLE BENDING STRESS BASED ON COMBINED STRESS ALLOWABLES IN B31.8 SECTION A842.223 WITH POISSONS EFFECT CONSIDERED:

$$\sigma_{C_{MAX}} = \sqrt{\sigma_H^2 - \sigma_L \sigma_H + \sigma_L^2} = 0.9 S_y = (0.9)(52{,}000\ \text{psi}) = 46{,}800\ \text{psi} = \text{COMBINED STRESS LIMIT}$$

$$\sigma_L = \frac{\sigma_H \pm \sqrt{(-\sigma_H^2) - (4)(\sigma_H^2 - \sigma_{C_{MAX}}^2)}}{2} = \frac{(23{,}060\ \text{psi}) \pm \sqrt{(-23{,}060\ \text{psi})^2 - 4((23{,}060\ \text{psi})^2 - (46{,}800\ \text{psi})^2}}{2}$$

$$= \left\{ \begin{array}{l} \sigma_{L_1} = 53{,}860\ \text{psi} \\ \sigma_{L_2} = -30{,}800\ \text{psi} \end{array} \right\} = \text{LONGITUDINAL STRESS COMPONENTS}$$

$$\sigma_{b2} = \left\{ \text{THE LESSER OF} \begin{array}{l} \left|\sigma_{L_1} - \sigma_P\right| = \left|(53{,}860\ \text{psi}) - (-6{,}918\ \text{psi})\right| \\ \left|\sigma_{L_2} - \sigma_P\right| = \left|(-30{,}800\ \text{psi}) - (-6{,}918\ \text{psi})\right| \end{array} \right\} = 23{,}882\ \text{psi}$$

$=$ AVAILABLE BENDING STRESS BASED ON VON MISES COMBINED STRESS ALLOWABLES

STATIC 2: FREE SPAN BEAM ANALYSIS BASED ON B31.8 LONGITUDINAL AND COMBINED STRESS ALLOWABLES (CONTINUED) (3 OF 3)

LOCAL VARIABLES (CONTINUED):

DETERMINE MAXIMUM SPAN USING BEAM FLEXURAL FORMULAS:

$$\sigma_b = \left\{ \text{THE LESSER OF} \begin{array}{l} \sigma_{b1} = 34{,}682 \text{ psi} \\ \sigma_{b2} = 23{,}882 \text{ psi} \end{array} \right\} = 23{,}882 \text{ psi} = \text{MAXIMUM AVAILABLE BENDING STRESS}$$

$$M = Z\sigma_b = \frac{wL^2}{10} = \text{MAXIMUM BENDING MOMENT FOR A PARTIALLY FIXED BEAM}^{2,3}$$

$$L = \sqrt{\frac{10 Z \sigma_b}{w}} = \sqrt{\frac{10 (130.2 \text{ in}^3)(23{,}882 \text{ psi})}{139.6 \text{ lbs/ft}} \left(\frac{1 \text{ ft}}{12 \text{ in}}\right)} = 136 \text{ ft} =$$

= MAXIMUM ALLOWABLE FREE SPAN LENGTH

REFERENCES:

1.) ASME B31.8-1995 EDITION, "GAS TRANSMISSION AND DISTRIBUTION PIPING SYSTEMS." AMERICAN SOCIETY OF MECHANICAL ENGINEERS. 1995. PP. 97-98.

2.) AVALLONE, E.A. AND BAUMEISTER, T. III, MARKS' STANDARD HANDBOOK FOR MECHANICAL ENGINEERS. MCGRAW-HILL BOOK COMPANY. NINTH EDITION. 1987. pg 5-24

3.) SHAH, B.C., WHITE, C.N. AND RIPPON, I.J. "DESIGN AND OPERATIONAL CONSIDERATIONS FOR UNSUPPORTED OFFSHORE PIPELINE SPANS." O.T.C. 5216. PROCEEDINGS FROM THE 18TH ANNUAL OFFSHORE TECHNOLOGY CONFERENCE. HOUSTON TX. 1986. pg. 5

STATIC 3: STATIC ANALYSIS FOR ELEVATED OBSTRUCTION INDUCED FREE SPANS (1 OF 3)

DESCRIPTION OF CALCULATIONS:

THIS METHOD IS TO BE USED IN LIEU OF MODULE STATIC 1 IF THE PIPELINE IS INDUCED BY AN ELEVATED OBSTRUCTION. THIS METHOD DEVELOPES A PROCEDURE, BASED A PROCEDURE BY MOUSELLI[1]. NON-DIMENSIONAL PARAMETERS ARE FIRST DETERMINED. THE GRAPHS PRESENTED IN FIGS. 3.24 AND 3.25 ARE CONVERTED TO EQUATIONS BY REGRESSION CURVE FITTING. AFTER DIMENSIONLESS SPANS ARE FOUND FOR $\beta = 0$ AND $\beta = 10$, THE DIMENSIONLESS SPAN IS FOUND FOR β_{ACTUAL} THROUGH LINEAR INTERPOLATION. THE MAXIMUM SPAN LENGTH CAN THEN BE CALCULATED FROM THE DIMENSIONLESS SPAN.

ASSUMPTIONS:

THERMAL EXPANSION EFFECTS ARE NEGLIGIBLE

PIPE IS GEOMETRICALLY SYMMETRICAL

PIPE IS FLOODED WITH SEAWATER

GLOBAL VARIABLES:

$D_o = 18.00''$ = PIPE OUTSIDE DIAMETER

$I = 1171.5\ in^4$ = MOMENT OF INERTIA OF PIPE CROSS SECTION

$E = 2.9 \times 10^7\ psi$ = YOUNG'S MODULUS

$w = 139.6\ \frac{LBS}{FT}$ = SUBMERGED WEIGHT OF PIPE

$S_y = 52{,}000\ psi$ = SPECIFIED MINIMUM YIELD STRENGTH

LOCAL VARIABLES:

DIMENSIONLESS PARAMETERS ARE FIRST DETERMINED. DIRECT SUBSTITUTION IS USED TO COMPUTE DIMENSIONLESS STRESS AND DIMENSIONLESS TENSION. DIMENSIONLESS ELEVATION AND DIMENSIONLESS SPAN IS DETERMINED USING EQUATIONS THAT ARE CURVE FIT TO MOUSELLI FIGS 3.24 AND 3.25, RESPECTIVELY. ACTUAL DIMENSIONLESS SPAN IS DETERMINED BY LINEAR INTERPOLATION BETWEEN $\beta = 0$ AND $\beta = 10$.[2]

STATIC 3: STATIC ANALYSIS FOR ELEVATED OBSTRUCTION INDUCED FREE SPANS (CONTINUED) (2 OF 3)

LOCAL VARIABLES (CONTINUED):

$\sigma_m = 0.8\, S_Y = (52{,}000\ psi)(0.8) = 41{,}600\ psi$ = MAXIMUM STRESS BASED ON B31.8 ALLOWABLES IN TABLE A842.22.

$c = D_o/2 = 18.00\,in/2 = 9.00\,in$ = PIPE OUTER RADIUS

$$L_c = \left(\frac{EI}{w}\right)^{1/3} = \left[\frac{(2.9\times10^7\ psi)(1171.5\ in^4)\left(\frac{1\ ft^2}{144\ in^2}\right)}{139.6\ lbm/ft.}\right]^{1/3} = 119.1\ ft$$ = CHARACTERISTIC LENGTH

$$\sigma_c = \frac{Ec}{L_c} = \left[\frac{(2.9\times10^7 psi)(9.0\ in)}{(119.1\ ft.)(12\ in/ft)}\right] = 182{,}619.6\ psi$$ = CHARACTERISTIC STRESS

$T = 0$ KIPS = PIPE AXIAL TENSION

$$\beta = \frac{T}{wL_c} = \frac{0\ KIPS}{(139.6\ lbs/ft)(119.1 ft)}\left(\frac{1000\ lbs}{1\ KIP}\right) = 0$$ = DIMENSIONLESS TENSION

$$\frac{\sigma_m}{\sigma_c} = \frac{41{,}600\ psi}{182{,}619.6\ psi} = 0.228$$ = DIMENSIONLESS STRESS

$$\frac{100\delta}{L_c} = 0.02323 + 1.251\left(\frac{\sigma_m}{\sigma_c}\right) + 52.18\left(\frac{\sigma_m}{\sigma_c}\right)^2 - 16.02\left(\frac{\sigma_m}{\sigma_c}\right)^3,\quad 0 \le \frac{\sigma_m}{\sigma_c} \le 0.405$$

$$= \frac{100\delta}{L_c} = 0.0232 + 1.251(0.228) + 52.18(0.228)^2 - 16.02(0.228)^3$$

$= 2.831$ = MAXIMUM ALLOWABLE DIMENSIONLESS ELEVATION DUE TO ELEVATED OBSTRUCTION

$\frac{100\delta}{L_c}$ CAN BE PLUGGED INTO THE EQUATION FIT FOR MOUSELL FIG 3.24[1]

$$\left.\frac{L}{L_c}\right|_{\beta=0} = \begin{cases} 0 + 5.667\left(\frac{100\delta}{L_c}\right) - 7.600\left(\frac{100\delta}{L_c}\right)^2 + 3.733\left(\frac{100\delta}{L_c}\right)^3, & 0 \le x \le 1 \\ 1.409 + 0.41239\left(\frac{100\delta}{L_c}\right) - 3.437\times10^{-2}\left(\frac{100\delta}{L_c}\right)^2 + 1.042\times10^{-3}\left(\frac{100\delta}{L_c}\right)^3, & 1 < x \le 7 \end{cases}$$

$$\left.\frac{L}{L_c}\right|_{\beta=10} = \begin{cases} 0 + 5.150\left(\frac{100\delta}{L_c}\right) - 5.100\left(\frac{100\delta}{L_c}\right)^2 + 2.000\left(\frac{100\delta}{L_c}\right)^3, & 0 \le x \le 1 \\ 1.609 + 0.4740\left(\frac{100\delta}{L_c}\right) - 3.437\times10^{-2}\left(\frac{100\delta}{L_c}\right)^2 + 1.042\times10^{-3}\left(\frac{100\delta}{L_c}\right)^3, & 1 < x \le 7 \end{cases}$$

STATIC 3: STATIC ANALYSIS FOR ELEVATED OBSTRUCTION INDUCED FREE SPANS (CONTINUED) (3 OF 3)

LOCAL VARIABLES (CONTINUED):

$$\left.\frac{L}{L_c}\right|_{\beta=0} = 1.409 + 0.4239(2.831) - 3.4137\times10^{-2}(2.831)^2 + 1.0412\times10^{-3}(2.831)^3 = 2.357$$

= DIMENSIONLESS SPAN AT $\beta = 0$

$$\left.\frac{L}{L_c}\right|_{\beta=10} = 1.609 + 0.4740(2.831) - 3.437\times10^{-2}(2.831)^2 + 1.042\times10^{-3}(2.831)^3 = 2.699$$

= DIMENSIONLESS SPAN AT $\beta = 10$.

SOLVE FOR DIMENSIONLESS SPAN BY LINEAR INTERPOLATION USING β_{ACTUAL}

$$\frac{L}{L_c} = \left(\frac{\beta-10}{0-10}\right)\left(\left.\frac{L}{L_c}\right|_{\beta=0} - \left.\frac{L}{L_c}\right|_{\beta=10}\right) + \left.\frac{L}{L_c}\right|_{\beta=10}$$

$$= \left(\frac{(0)-10}{0-10}\right)\left((2.357-(2.699)\right) + 2.699$$

$= 2.357$ = DIMENSIONLESS SPAN AT β_{ACTUAL}

$$L = \left(\frac{L}{L_c}\right)(L_c) = (2.357)(119.1\ FT) = 280.7\ FT.$$

REFERENCES:

1.) MOUSELLI, A.H. OFFSHORE PIPELINE DESIGN, ANALYSIS, AND METHODS. PENNWELL PUBLISHING CO. 1981.

2.) ASME B31.8 – 1995 EDITION. "GAS TRANSMISSION AND DISTRIBUTION PIPING SYSTEMS." AMERICAN SOCIETY OF MECHANICAL ENGINEERS. 1995.

VORTEX 1: GENERAL VORTEX INDUCED VIBRATION ANALYSIS (1 OF 2)

DESCRIPTION OF CALCULATIONS:

THIS METHOD VORTEX SHEDDING FREQUENCY AND THE NATURAL FREQUENCY OF A FREE SPAN TO DETERMINE THE MAXIMUM ALLOWABLE FREE SPAN LENGTH. VORTEX INDUCED OSCILLATION EQUATIONS ARE BASED ON THOSE DEVELOPED BY NIELSEN AND GRAVESEN[1] AND MOUSELLI[3].

ASSUMPTIONS:

IN LINE VORTEX INDUCED VIBRATIONS ARE NOT CONSIDERED

PIPE IS FLOODED WITH SEA WATER

PIPE IS PARTIALLY FIXED AT EACH END

FLOW INCIDENT ANGLE ON PIPE IS 90°

GLOBAL VARIABLES:

D_{TOT} = 21.00 IN = PIPE OUTSIDE DIAMETER WITH CONCRETE WEIGHT COATING

f_s = 0.375 HZ = VORTEX SHEDDING FREQUENCY

M = 13.90 SLUGS/FT = DYNAMIC MASS OF SUBMERGED PIPE

S_T = 0.219 = STROUHAL NUMBER

$E = 2.9\times10^7$ PSI = YOUNG'S MODULUS

I = 1171.5 IN⁴ = MOMENT OF INERTIA OF PIPE CROSS SECTION

C = 2.55 = FREE SPAN END FIXITY CONSTANT

e = 3 FT. = GAP BETWEEN SEAFLOOR AND PIPE

LOCAL VARIABLES:

$$f_s \le 0.7 f_n \quad {}^{1}$$

$$f_n = \frac{K\pi}{2L^2}\sqrt{\frac{EI}{M}}$$ = FREE SPAN NATURAL FREQUENCY

VORTEXI: GENERAL VORTEX INDUCE VIBRATION ANALYSIS (2 OF 2)

LOCAL VARIABLES (CONTINUED):

SUBSTITUTION YIELDS[2,3]:

$$\frac{f_s}{0.7} \leq \frac{K\pi}{2L^2}\sqrt{\frac{EI}{m}}$$

$$\frac{K\pi}{2} = C = 2.55$$

SUBSTITUTING FOR L YIELDS THE FOLLOWING LIMITING EQUATION:

$$L = \left(\frac{0.7C}{f_s}\left(\frac{EI}{m}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}$$

$$L = \left[\left(\frac{(0.7)(2.54)}{0.375\ 1/s}\right)\left(\frac{(2.9\times10^7\ \frac{LBS}{IN^2})(1171.5\ IN^4)\left(\frac{1\ FT^2}{144\ IN^2}\right)}{(13.90\ SLUGS/FT)}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}$$

$= 139.5$ FT $=$ MAXIMUM ALLOWABLE FREE SPAN LENGTH

REFERENCES

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VORTEX2: CROSS FLOW VORTEX INDUCED VIBRATION ANALYSIS (1 OF 4)

DESCRIPTION OF CALCULATIONS:

THIS ANALYSIS METHOD IS BASED ON THE RESEARCH FROM THE MULTISPAN PROJECT.[3] THE METHOD CALCULATES THE MAXIMUM ALLOWABLE FREE SPAN LENGTH BY PREVENTING THE ONSET OF CROSS FLOW VORTEX INDUCED VIBRATIONS. NATURAL FREQUENCY OF THE FREE SPAN IS DETERMINED BY USING THE METHOD OUTLINED IN NIELSEN AND GRAVESEN.[7]

ASSUMPTIONS:

ZERO AXIAL PIPE TENSION

PIPE IS FLOODED WITH SEA WATER

FLOW INCIDENT ANGLE IS 90° WITH RESPECT TO PIPE

TURBULENCE IS LESS THAN 8%

NO EXTREAM CURRENT VARIABILITY

INFINITELY LONG CYLINDER IS ASSUMED FOR PIPELINE

GLOBAL VARIABLES:

D_o = 18.00 IN = PIPE OUTSIDE DIAMETER

t = 0.562 IN = PIPE WALL THICKNESS

t_c = 1.50 IN = CONCRETE WEIGHT COATING THICKNESS

D_i = 16.876 IN = PIPE INSIDE DIAMETER

D_{TOT} = 21.00 IN = TOTAL PIPE OUTSIDE DIAMETER WITH CONCRETE COATING

E = 2.9×10^7 PSI = YOUNGS MODULUS

I = 1171.5 IN^4 = MOMENT OF INERTIA OF PIPE CROSS SECTION

C = 2.55 = FREE SPAN FIXITY CONSTANT

U = 3.0 FT/S = SEA CURRENT VELOCITY FOR A 100 YR. RETURN PERIOD STORM

S_y = 52,000 PSI = SPECIFIED MINIMUM YIELD STRENGTH OF PIPE

e = 3 FT = GAP BETWEEN SEAFLOOR AND PIPELINE

S_t = 0.219 = STROUHAL NUMBER

w = 139.6 LBS/FT = SUBMERGED WEIGHT OF PIPE

VORTEX 2: CROSS FLOW VORTEX INDUCED VIBRATION ANALYSIS (CONTINUED) (2 OF 4)

LOCAL VARIABLES

THE DAMPING RATIO IS DETERMINED BY SOLVING THE LOGARITHMIC DECREMENT EQUATION FOR ζ. THE RECOMMENDED VALUE FOR DAMPING RATIO WAS EVALUATED BY USING THE VALUE OF 0.05 FOR THE LOGARITHMIC DECREMENT[1].

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = 0.05 = \text{LOGARITHMIC DECREMENT}$$

$$\zeta = \pi\left(\frac{\sqrt{\left(\frac{\delta}{\pi}\right)^2+1} - 1}{\delta}\right)$$

$$= \pi\left(\frac{\sqrt{\left(\frac{(0.05)}{\pi}\right)^2+1} - 1}{0.05}\right) = 0.008 = \text{DAMPING RATIO}$$

THE STABILITY PARAMETER IS A FUNCTION OF SPECIFIC WEIGHT, ADDED MASS COEFFICIENT[2], AND STROUHAL NUMBER[8].

$$w_w = \frac{\pi}{4} D_{TOT}^2 \rho_{ow} = \frac{\pi}{4}\left(\frac{21.00\text{ IN}}{12\text{ IN/FT}}\right)^2 \left(64\text{ LBS/FT}^3\right) = 153.9\text{ LBS/FT}$$

= WEIGHT OF DISPLACED WATER

$$w_s = \frac{\pi}{4}\left[\left(D_o^2 - D_i^2\right)\rho_{os} + \left(D_{TOT}^2 - D_o^2\right)\rho_{oc}\right]$$

$$= \frac{\pi}{4}\left[\left((18.00\text{ IN})^2 - (16.876\text{ IN})^2\right)\left(490\text{ LBS/FT}^3\right) + \left((21.00\text{ IN})^2 - (18.00\text{ IN})^2\right)\left(140\frac{\text{LBS}}{\text{FT}^3}\right)\right]\left(\frac{1\text{ FT}^2}{144\text{ IN}^2}\right)$$

$= 194.1$ LBS/FT = DRY WEIGHT OF PIPE WITH CONCRETE COATING

$$\frac{\rho_s}{\rho} = \frac{w_s}{w} = \frac{194.1\text{ LBS/FT}}{153.9\text{ LBS/FT}} = 1.26 = \text{SPECIFIC WEIGHT OF PIPE}$$

$C_m = 1.0$ = ADDED MASS COEFFICIENT[2]

$$S_T = 0.27 - 0.03\left(\frac{e}{D_{TOT}}\right) = 0.27 - 0.03\left(\frac{3\text{ FT}}{21.00\text{ IN}}\right)\left(\frac{12\text{ IN}}{1\text{ FT}}\right) = 0.219$$

= STROUHAL NUMBER[8]

$$K_s = \pi^2\zeta\left(\frac{\rho_s}{\rho} - C_m\right) = \pi^2(0.008)(1.26-1.0) = 0.020 = \text{STABILITY PARAMETER}^{8}$$

VORTEX 2: CROSS FLOW VORTEX INDUCED VIBRATION ANALYSIS (CONTINUED) (3 OF 4)

LOCAL VARIABLES (CONTINUED)

THE REDUCED VELOCITY IS A FUNCTION OF PIPE DIAMETER AND SEA CURRENT VELOCITY. 100 YR. RETURN PERIOD STORM DATA IS USED TO EVALUATE REDUCED VELOCITY. PARTIAL SAFETY FACTORS ARE APPLIED TO THE EQUATION.

$$V_{R,\,ONSET\ CROSS\text{-}FLOW} = \sqrt{\frac{\pi^3\left(\frac{\rho_s}{\rho}+C_m\right)}{1.5+S_t^2\left(\pi^3\left(\frac{\rho_s}{\rho}+C_m\right)-1.5K_s^2\right)}}$$

$$= \sqrt{\frac{\pi^3(1.26+1.0)}{1.5+(0.219)^2\left(\pi^3(1.26+1.0)-1.5(0.0205)^2\right)}}$$

$= 3.797 =$ FREE SPAN REDUCED VELOCITY

$\gamma_T = 2.0 =$ NORMAL SAFETY CLASS COEFFICIENT [8]

$\psi_D = 1.0$ [6]

$\psi_K = 1.0$ [3]

$\psi_L = 1.0$ [5]

$U = 3.0\ FT/s =$ 100 YR. RETURN PERIOD STORM CURRENT (ARBITRARY)

$$f_n \geq \frac{U}{V_{R,\,ONSET\ CROSS\text{-}FLOW}\, D_{OT}}\gamma_T\psi_D\psi_K\psi_L = \frac{3.0\ FT/s}{(3.797)\left(\frac{21.00\ IN}{12\ IN/FT}\right)}(2.0)(1.0)(1.0)(1.0)$$ [3]

$= 0.903\ Hz =$ LIMITING CRITERIA FOR THE ONSET OF CROSS FLOW VIBRATIONS

$$f_n = \frac{K\pi}{2L^2}\sqrt{\frac{EI}{M}} = \frac{C}{L^2}\sqrt{\frac{EI}{M}}$$ = FREE SPAN NATURAL FREQUENCY [7]

SUBSTITUTE LIMITING CRITERIA INTO NATURAL FREQUENCY EQUATION

$$L = \sqrt{\frac{C\sqrt{EI/M}}{f_n}} = \sqrt{\frac{2.55\sqrt{(2.9\times10^7\ PSI)(1171.5\ IN^4)\left(\frac{32.2\ FT}{S^2}\right)\left(\frac{1\ FT^2}{144\ IN^2}\right)\Big/(13.89\ SLUG/FT)(32.2\ LBS/SLUG)}}{0.903\ Hz}}$$

$= 107.7$ FT. = MAXIMUM ALLOWABLE FREE SPAN LENGTH

VORTEX 2: CROSS FLOW VORTEX INDUCED VIBRATION ANALYSIS (CONTINUED) (4 OF 4)

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